Unsmoothing Returns of Illiquid Assets

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Abstract

Funds that invest in illiquid assets report returns with spurious autocorrelation because their underlying assets trade infrequently. These returns need to be unsmoothed to properly measure risk exposures. We show that traditional unsmoothing techniques do not fully unsmooth the systematic component of returns, thereby understating risk exposures and overstating risk-adjusted performance. We then propose a novel return unsmoothing method to address this issue and apply it to hedge funds and commercial real estate funds. In doing so, we find a substantial improvement in the measurement of risk exposures and risk-adjusted performance. Furthermore, the improvement is greater for more illiquid funds.

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Introduction

The market size of intermediaries investing in illiquid assets has grown dramatically over the last two decades.[1] However, there is a lot we do not know about their risks and performance due to the difficulty in measuring these quantities with standard techniques. Specifically, reported returns partially reflect past changes in economic values when reported and economic values differ due to infrequent trading and sporadic marking to market. This smoothing effect creates spurious return autocorrelation and invalidates traditional measures of risk and performance (betas and alpha). The crux of the problem is that we only observe reported (or smoothed) returns, while we need economic (or unsmoothed) returns to evaluate risk and risk-adjusted performance.

Prior research provides different ways to recover economic return estimates by unsmoothing observed returns (e.g., Geltner (1993) and Getmansky, Lo, and Makarov (2004)). In this paper, we argue that while previous techniques represent an important first step in measuring the risks of illiquid assets, they do not fully unsmooth the systematic component of returns, and thus understate the importance of risk factors in explaining illiquid assets returns. We then provide an adjustment to return unsmoothing techniques to deal with this issue and apply our novel methodology to hedge funds and Commercial Real Estate (CRE) funds, demonstrating its usefulness in measuring the risk exposures and risk-adjusted performance of illiquid assets. Our main finding is that systematic risk (and risk-adjusted performance) is better measured when returns are unsmoothed using our new return unsmoothing method.

The basic idea behind return unsmoothing methods is simple. These techniques assume observed returns are weighted averages of current and past economic returns, estimate these weights, and use them to recover estimates for economic returns, which are otherwise unobservable.

[1]For instance, a report by the PwC Asset and Wealth Management Research Center shows a growth of roughly 400% (from \$2.5 trillion to \$10.1 trillion) in assets under management for alternative investments (which are typically illiquid) from 2004 to 2016 (see pwc (2018)). The same report indicates that the hedge fund industry (the focus of a substantial part of our empirical analysis) has grown from \$1.0 trillion to \$3.3 trillion over the same period.

Traditional return unsmoothing techniques perform well when evaluating fund-level returns. For instance, applying Getmansky, Lo, and Makarov (2004)'s method, which is ubiquitous in the hedge fund literature, to unsmooth returns of hedge funds focused on relative value strategies reduces the average monthly return autocorrelation from 0.30 to 0.01. Similarly, using Geltner (1993)'s technique, which is common in the real estate literature, to unsmooth returns of CRE funds reduces the average quarterly return autocorrelation from 0.72 to 0.02. These results suggest unsmoothed returns better reflect the contemporaneous change in the true value of their underlying assets.

Despite this success, we find that aggregating unsmoothed returns produces index-level returns that display significant autocorrelation. For instance, the monthly autocorrelation of the relative value strategy index constructed from unsmoothed hedge fund returns is still 0.29. Similarly, the quarterly autocorrelation of the CRE index constructed from unsmoothed fund returns is still 0.54. These results indicate that the systematic component of fund-level returns is not fully unsmoothed based on traditional methods, making it difficult to properly measure fund risk exposures.

To deal with this issue, we propose a simple adjustment to traditional return unsmoothing methods. We assume that the observed returns of each fund are weighted averages of current and past economic returns on both the fund and the aggregate of similar funds. We then show how to use a 3-step procedure to estimate the weights and obtain economic return estimates.

The 3-step unsmoothing method we propose collapses to traditional (or 1-step) unsmoothing if observed fund returns do not directly reflect lagged aggregate returns of similar funds (an underlying assumption in Getmansky, Lo, and Makarov (2004) and Geltner (1993)). However, the data do not support this assumption. Figure 1 shows (for CRE funds and hedge funds sorted on strategy illiquidity) average (normalized) coefficients from regressing fund returns on lagged fund returns and lagged aggregate returns of similar funds. When estimating univariate regressions in which only lagged fund returns are included, the coefficients are strong for relatively illiquid funds. Nevertheless, when lagged aggregate returns are

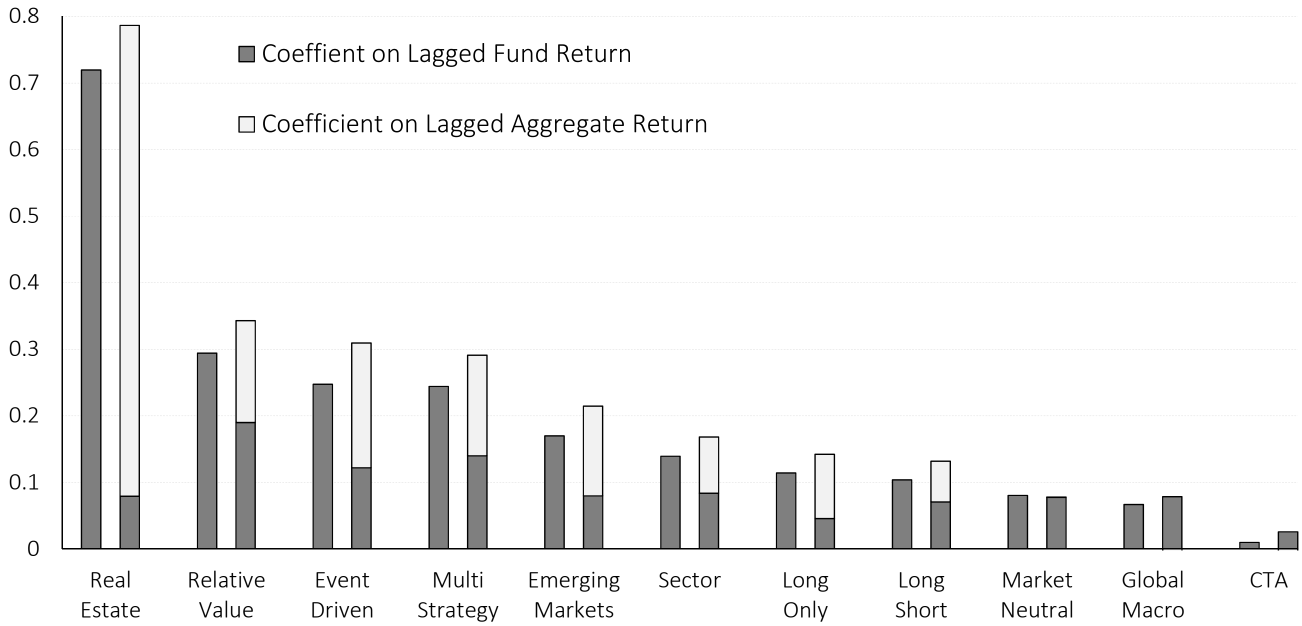

Figure 1
Regressing Fund Returns on Lagged Fund and Aggregate Returns

The figure plots coefficients from regressing (observed) fund returns on lagged (observed) fund and aggregate returns. The first bar for each fund category is based on a univariate regression while the second bar relies on a bivariate regression. Observed returns are normalized to have a unit standard deviation so that the coefficients are comparable. The first two bars reflect real estate funds (sample from 1994 through 2017) while the other bars reflect different hedge fund strategies (sample from January 1995 to December 2016). See Sections 2.1 and 3.2 for further empirical details.

also included in the regressions, the coefficients on lagged fund returns tend to decline, with the coefficients on lagged aggregate returns capturing a substantial portion of the smoothing effect. These results point to a mispecification in 1-step unsmoothing methods, and thus we apply our 3-step unsmoothing procedure to hedge funds and CRE funds.

In the case of hedge funds, autocorrelation effectively disappears both at the fund-level and strategy-level, which suggests that 3-step unsmoothing is better able to unsmooth the systematic component of hedge fund returns than 1-step unsmoothing. Motivated by this finding, we explore the implications of our methodology to the measurement of risk exposures and risk-adjusted performance of hedge funds.

We perform two main exercises using hedge fund returns. In the first exercise, we sort funds into three groups based on the liquidity of their underlying strategy, and apply 1-step and 3-step unsmoothing to funds in each of these groups. We then measure, for each group, average fund volatility as well as risk and risk-adjusted performance based on a standard factor model used in the hedge fund literature (the FH 8-Factor model that builds on Fung and Hsieh (2001)). We find that volatility substantially increases after unsmoothing returns, but the increase is roughly the same whether we use 1-step or 3-step unsmoothing. In contrast, 3-step unsmoothing produces economic returns that comove more strongly with FH risk factors (relative to 1-step unsmoothing) and display lower alphas as a consequence. The aforementioned results hold only in the low and mid liquidity groups. Unsmoothing returns of funds in the high liquidity group has no effect on volatility, R^2s, and alphas. This result indicates that unsmoothing techniques do not produce unintended distortions in the estimates of economic returns for liquid assets.

In the second exercise, we repeat the analysis described above separately for funds in each major hedge fund strategy category and find similar results. However, grouping funds based on their underlying strategies allows us to study funds exposed to similar risks, and thus to explore how our 3-step unsmoothing technique improves the measurement of systematic risk exposures. We find that our 3-step unsmoothing tends to change risk exposure estimates in ways consistent with economic logic. For example, after unsmoothing returns using our 3-step method, the exposure of emerging market funds to the emerging market risk factor strongly increases, while other risk exposures of emerging market funds display little change.

Turning to CRE funds, the overall results are similar to what we obtain with hedge funds (except that the method still leaves a portion of systematic returns unsmoothed). However, the degree to which the 3-step unsmooth process improves upon 1-step unsmoothing is much higher given the extreme illiquidity of real estate assets. For instance, the average beta of private CRE funds to the public real estate market increases from 0.04 to 0.56, effectively driving the 5% annual alpha of CRE funds (measured with observed returns) to zero after 3-step unsmoothing.

In summary, we develop a 3-step process to improve upon traditional return unsmoothing techniques for illiquid assets in order to better estimate their systematic risk exposures. We then apply our new method to hedge funds, finding that the measurement of risk exposures and risk-adjusted performance substantially improves after the adjustment. Finally, we perform a similar analysis based on CRE funds given their high degree of illiquidity and find that results are even more pronounced for these funds.

Our paper provides a general contribution to the literature on illiquid assets as it develops a simple way to recover economic return estimates from observed returns in order to measure their risk and risk-adjusted performance. Several papers in this body of literature attempt to measure the illiquidity premium (e.g., Aragon (2007), Khandani and Lo (2011), and Barth and Monin (2018)). Our contribution is particularly important in this area because unsmoothing returns is an essential part of measuring the illiquidity premium. Without properly unsmoothing returns, any attempt to measure this premium would not correctly control for exposure to other sources of risk and, as a consequence, would attribute the premium associated with other risk factors to illiquidity.

Our 3-step process to improve upon Getmansky, Lo, and Makarov (2004) is a significant contribution to the hedge fund literature as it is standard practice to apply the Getmansky, Lo, and Makarov (2004) technique to unsmooth returns before studying hedge funds (e.g., Kosowski, Naik, and Teo (2007), Fung et al. (2008), Patton (2008), Agarwal, Daniel, and Naik (2009), Teo (2009), Kang et al. (2010), Jagannathan, Malakhov, and Novikov (2010), Titman and Tiu (2011), Teo (2011), Avramov et al. (2011), Aragon and Nanda (2011), Billio et al. (2012), Patton and Ramadorai (2013), Bollen (2013), Berzins, Liu, and Trzcinka (2013), Li, Xu, and Zhang (2016), Agarwal, Ruenzi, and Weigert (2017), Gao, Gao, and Song (2018), and Agarwal, Green, and Ren (2018)).[2,3] Our empirical analysis further adds

[2]Some papers use the alternative approach of including lags of risk factors in the factor regressions (e.g., Chen (2011), Bali, Brown, and Caglayan (2012), and Cao et al. (2013)). While useful as a quick solution to the return autocorrelation problem, this approach does not provide a way to recover economic returns and it dramatically increases the number of parameters to be estimated in the factor regressions, which is an important limitation as investors often face the problem of measuring multiple risk exposures based on a relatively short time-series.

[3]While we interpret our results from the lens of illiquidity, misreporting can also induce smoothed returns (Bollen and Pool (2008, 2009) and Aragon and Nanda (2017)). We do not attempt to disentangle the two

to previous papers in the hedge fund literature by demonstrating that hedge fund alphas are lower than previously recognized, once systematic risk is properly measured using our 3-step unsmoothing method.

We also contribute to the real estate literature because we show that our unsmoothing technique can be used to improve upon the autoregressive unsmoothing method in Geltner (1993), which is commonly applied in the real estate literature (e.g., Fisher, Geltner, and Webb (1994), Barkham and Geltner (1995), Corgel et al. (1999), Pagliari Jr, Scherer, and Monopoli (2005), Rehring (2012), and Pagliari Jr (2017)).

Our improvement upon traditional unsmoothing techniques reaches beyond the hedge fund and real estate literatures, however. For instance, unsmoothing methods have been applied to other types of illiquid funds such as private equity, venture capital, and bond mutual funds (Chen, Ferson, and Peters (2010) and Ang et al. (2018)), to highly illiquid assets such as collectible stamps and art investments (Dimson and Spaenjers (2011) and Campbell (2008)), and even to unsmooth other economic series such as aggregate consumption (see Kroencke (2017)).

The rest of this paper is organized as follows. Section 1 introduces the traditional, 1-step, return unsmoothing framework and develops our 3-step unsmoothing process to improve upon it; Section 2 applies 1-step and 3-step unsmoothing to hedge fund returns and demonstrates that the latter improves upon the former in measuring risk exposures and risk-adjusted performance; Section 3 extends the analysis to CRE funds; and Section 4 concludes. The Internet Appendix provides supplementary results.

1 A New Unsmoothing Method

Academics and practitioners rely primarily on two methods in order to obtain economic (or unsmoothed) returns, R_t, from observed (or smoothed) returns, R_t^o. These methods can be

sources of return smoothness because, when considering the perspective of an investor or econometrician attempting to estimate economic returns, the degree of return smoothness is the relevant variable, not the source of smoothness. Moreover, Cassar and Gerakos (2011) and Cao et al. (2017) find that asset illiquidity is the major driver of autocorrelation in hedge fund returns.

used to unsmooth illiquid asset returns or returns of funds that invest in illiquid assets, but we refer to returns as belonging to funds for concreteness since, in our empirical analysis, we unsmooth returns of hedge funds and commercial real estate funds.

Both unsmoothing methods assume R_t^o is a weighted average of current and past R_t, but the two techniques differ in how the weights are specified. The first method, developed by Getmansky, Lo, and Makarov (2004), leaves weights unconstrained, but requires a finite number of smoothing lags (we refer to this framework as MA unsmoothing as it implies a moving average time-series process for R_t^o). The second method, developed by Geltner (1993), allows for an infinite number of smoothing lags, but constrains weights to decay exponentially (we refer to this framework as AR unsmoothing as it implies an autoregressive time-series process for R_t^o). The former method is mainly used in the hedge fund literature while the latter is applied more generally to unsmooth returns of highly illiquid assets and its most common application is in the real estate literature.

We refer to the Getmansky, Lo, and Makarov (2004) (MA) and Geltner (1993) (AR) methods generally as "1-step unsmoothing" and develop a 3-step approach (for both MA and AR) that improves upon 1-step unsmoothing methods. Subsection 1.1 details the 1-step MA unsmoothing method; Subsection 1.2 demonstrates that it does not fully unsmooth the systematic portion of returns; and Subsection 1.3 develops our 3-step MA unsmoothing technique to address this issue. The description of the AR unsmoothing framework is provided in Section 3, where we also unsmooth returns of commercial real estate funds.

1.1 The 1-step MA Unsmoothing Method

Table 1 provides the basic characteristics of the hedge funds we study (the data sources and sample construction are detailed in Subsection 2.1). There are 10 different hedge fund strategies (sorted by the average fund-level 1st order return autocorrelation coefficient) with a total of 4,827 funds and an average of 89 monthly returns per fund. Hedge funds display (annualized) average excess returns varying from 2.3% to 5.7% and (annualized) Sharpe ratios varying from 0.22 to 0.60, with more illiquid funds displaying higher Sharpe ratios.

It is well known that some hedge fund strategies rely on illiquid assets, and thus the observed returns of hedge funds may be smoothed. To deal with this issue, Getmansky, Lo, and Makarov (2004) (henceforth GLM) propose a method to unsmooth hedge fund returns. GLM assume the observed return of fund j at time t is given by (see original paper for the economic motivation):[4]

$$R^o_{j,t} = \theta_j^{(0)} \cdot R_{j,t} \ + \ \theta_j^{(1)} \cdot R_{j,t-1} \ + ... + \ \theta_j^{(H)} \cdot R_{j,t-H_j} \tag{1}$$

$$= \mu_j \ + \Sigma_{h=0}^{H} \, \theta_j^{(h)} \cdot \eta_{j,t-h} \tag{2}$$

where θs represent the smoothing weights with $\Sigma_{h=0}^{H}\theta_j^{(h)} = 1$ and the second equality follows from GLM's assumption that $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$.

The first equality represents the economic assumption that the observed fund return is a weighted average of the fund's economic returns over the most recent $H + 1$ periods, including the current period. The second equality is an econometric implication that says that, under the given assumption, the observed fund returns follow a Moving Average process of order H, MA(H).

Given Equation 2, we can recover economic returns by estimating an MA(H) process for observed returns, $R^o_{j,t}$, extracting the estimated residuals, $\eta_{j,t}$, and adding the estimated expected return, $R_{j,t} = \mu_j + \eta_{j,t}$. GLM also provide the basic steps to estimate θs by maximum likelihood under the added parametric assumption that $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$.[5] This procedure is used by several papers in the hedge fund literature to unsmooth returns (e.g., Kosowski, Naik, and Teo (2007), Fung et al. (2008), Patton (2008), Agarwal, Daniel, and Naik (2009), Teo (2009), Kang et al. (2010), Jagannathan, Malakhov, and Novikov (2010), Titman and Tiu (2011), Teo (2011), Avramov et al. (2011), Aragon and Nanda (2011), Billio et al. (2012),

[4] The fact that H does not depend on j simplifies the notation but does not imply the number of MA lags is not fund dependent. Specifically, letting H_j represent the number of MA lags with non-zero weight for fund j, we define $H = max(H_j)$ and set $\theta_j^{(h)} = 0$ for any $h > H_j$.

[5] The method is almost identical to the one used in most statistical/econometric packages. The only difference is that statistical/econometric packages tend to impose the normalization $\theta_j^{(0)} = 1$ as opposed to $\Sigma_{h=0}^{H}\theta_j^{(h)} = 1$. To translate from the first to the second normalization, one needs to divide the coefficients estimated by the package (and multiple the estimated residuals) by $1 + \theta_j^{(1)} + ... + \theta_j^{(H)}$.

Patton and Ramadorai (2013), Bollen (2013), Berzins, Liu, and Trzcinka (2013), Li, Xu, and Zhang (2016), Agarwal, Ruenzi, and Weigert (2017), Gao, Gao, and Song (2018), and Agarwal, Green, and Ren (2018)).

We apply the 1-step MA unsmoothing to each hedge fund in our dataset using the AIC criterium to select the number of MA lags (allowing for H from 0 to 3 months). Table 2 contains the (average) autocorrelations (at 1, 2, 3, and 4 lags) for hedge fund returns (observed and unsmoothed) as well as the % of funds with a significant autocorrelation at 10% level. Observed returns display relatively high autocorrelations. For instance, relative value funds have average 1st order autocorrelation of 0.30, with 65.1% of these funds displaying statistically significant autocorrelations. After 1-step MA unsmoothing, average autocorrelations are basically zero at all lags and the % of funds displaying statistically significant autocorrelations is in line with the statistical error of the test.

The results indicate that 1-step MA unsmoothing produces economic returns that are largely unsmoothed at the fund level. This correction is important to properly analyse hedge funds because smoothed returns understate volatilities and betas, and thus overstate Sharpe ratios and alphas, as demonstrated by GLM.

1.2 Implications to Aggregate Fund Returns

Under the central assumption of unsmoothing methods, we should also observe strategy index returns that are not autocorrelated. Specifically, for any set of time-invariant weights, w_j, the assumption that $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$ implies:

$$\begin{aligned} \overline{R}_t &\equiv \Sigma_{j=1}^{J} w_j \cdot R_{j,t} \\ &= \Sigma_{j=1}^{J} w_j \cdot \mu_j \ + \ \Sigma_{j=1}^{J} w_{j,t} \cdot \eta_{j,t} \\ &= \overline{\mu} \ + \ \overline{\eta}_t \end{aligned} \tag{3}$$

where $\overline{\eta}_t \sim IID$, which means that $\mathbb{E}_{t-1}[\overline{R}_t] = \overline{\mu}$ (i.e., aggregate returns are unpredictable, and thus they should not be autocorrelated).

Table 3 shows autocorrelations for each (equal-weighted) strategy index. Reported returns display quite high autocorrelations. For instance, the relative value strategy has a 1st order autocorrelation of 0.51 (statistically significant at 1%). The autocorrelation coefficients remain high after aggregating unsmoothed returns. For instance, the relative value strategy still has an autocorrelation of 0.29 (statistically significant at 1%) after 1-step MA unsmoothing.

The results suggest that 1-step unsmoothing delivers strategy indexes with substantial autocorrelation, which indicates that the approach used by the previous literature to unsmooth returns does not fully unsmooth the systematic component of returns. This result is important because if the systematic portion of returns is not fully unsmoothed, then betas estimated after traditional unsmoothing will be understated and alphas overstated, which has non-trivial implications for performance analysis.

1.3 The 3-step MA Unsmoothing Method

We develop a 3-step unsmoothing procedure to addresses the issue raised in the previous subsection. The basic idea is to obtain unsmoothed returns as the sum of unsmoothed aggregate returns and unsmoothed excess returns, where the aggregate and excess returns are unsmoothed separately.

We refer to the "aggregate" of a generic variable, $y_{j,t}$, as $\overline{y}_t = \Sigma_{j=1}^{J} w_j \cdot y_{j,t}$ and its "excess" as $y_{j,t} - \overline{y}_t$, where w_j are arbitrary (but time-invariant) weights with $\Sigma_{j=1}^{J} w_j = 1$. Moreover, we keep the total number of funds, J, constant over time while developing our aggregation results.

This subsection relies on the fact that, for arbitrary variables x_j and $y_{j,t}$, we have:[6]

$$\begin{aligned} \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{j,t} &= \overline{x} \cdot \overline{y}_t \; + \; \Sigma_{j=1}^{J} w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) \\ &= \overline{x} \cdot \overline{y}_t \; + \; \widehat{Cov}(x_j, y_{j,t}) \end{aligned} \tag{4}$$

[6]This result is a generalization of the typical covariance decomposition for the case of non-equal weights:

$$\begin{aligned} \Sigma_{j=1}^{J} w_j \cdot (x_j - \overline{x}) \cdot (y_{j,t} - \overline{y}_t) &= \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{t,j} - \overline{y}_t \cdot \Sigma_{j=1}^{J} w_j \cdot x_j - \overline{x} \cdot \Sigma_{j=1}^{J} w_j \cdot y_{t,j} + \overline{x} \cdot \overline{y}_t \cdot \Sigma_{j=1}^{J} w_j \\ &= \Sigma_{j=1}^{J} w_j \cdot x_j \cdot y_{t,j} - \overline{x} \cdot \overline{y}_t \end{aligned}$$

We generalize the underlying assumption in Equation 1 so that aggregate economic returns can be directly used in the smoothing process:[7]

$$R^o_{j,t} = \Sigma_{h=0}^{H}\ \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h}\ +\ \Sigma_{h=0}^{L}\ \pi_j^{(h)} \cdot \overline{R}_{t-h} \tag{5}$$

$$= \mu_j\ +\ \Sigma_{h=0}^{H}\ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h}\ +\ \Sigma_{h=0}^{L}\ \pi_j^{(h)} \cdot \overline{\eta}_{t-h} \tag{6}$$

where $\overline{R}_t = \Sigma_{j=1}^{J} w_j \cdot R_{j,t}$ are aggregate returns, $\widetilde{R}_j = R_{j,t} - \overline{R}_t$ are excess returns, $\overline{\eta}$ and $\widetilde{\eta}$ are the respective shocks, and the weights add to one, $\Sigma_{h=0}^{H}\phi_j^{(h)} = \Sigma_{h=0}^{L}\pi_j^{(h)} = 1$.

In GLM, the weights on past economic returns, θs, add to one to assure that information is eventually incorporated into observed prices. Our restriction, $\Sigma_{h=0}^{H}\phi_j^{(h)} = \Sigma_{h=0}^{L}\pi_j^{(h)} = 1$, has the same effect.

Aggregating Equation 6 yields:

$$\begin{aligned}\overline{R}^o_t &= \overline{\mu}\ +\ \overline{\pi}^{(0)} \cdot \overline{\eta}_t\ +\ \overline{\pi}^{(1)} \cdot \overline{\eta}_{t-1}\ + ... +\ \overline{\pi}^{(L)} \cdot \overline{\eta}_{t-\overline{H}} \\ &\quad + \widehat{Cov}(\phi_j^{(0)}, \widetilde{\eta}_{j,t})\ +\ \widehat{Cov}(\phi_j^{(1)}, \widetilde{\eta}_{j,t-1})\ + ... +\ \widehat{Cov}(\phi_j^{(H)}, \widetilde{\eta}_{j,t-H}) \\ &\approx \overline{\mu}\ +\ \Sigma_{h=0}^{L}\ \overline{\pi}^{(h)} \cdot \overline{\eta}_{t-h}\end{aligned} \tag{7}$$

where the first equality relies on Equation 4 and the second equality is based on a large sample approximation that uses $\underset{J\to\infty}{Plim}\ \widehat{Cov}(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = Cov(\phi_j^{(h)}, \widetilde{\eta}_{j,t-h}) = 0$. The restriction $\Sigma_{h=0}^{L}\pi_j^{(h)} = 1$ assures the aggregate moving average parameters satisfy $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = 1$ so that aggregate information is eventually incorporated into aggregate prices.[8]

Subtracting Equation 7 from Equation 6, we have observed excess returns:

$$\begin{aligned}\widetilde{R}^o_{j,t} &= \Sigma_{h=0}^{H}\ \phi_j^{(h)} \cdot \widetilde{R}_{j,t-h}\ +\ \Sigma_{h=0}^{L}\ \psi_j^{(h)} \cdot \overline{R}_{t-h} \\ &= \widetilde{\mu}_j\ +\ \Sigma_{h=0}^{H}\ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h}\ +\ \Sigma_{h=0}^{L}\ \psi_j^{(h)} \cdot \overline{\eta}_{t-h}\end{aligned} \tag{8}$$

[7]This smoothing process reduces to Equation 1 (the 1-step unsmoothing process in GLM) if we set $\pi_j^{(h)} = \phi_j^{(h)} = \theta_j^{(h)}$. Moreover, as in the 1-step method, the fact that H and L do not depend on j simplifies the notation but does not imply the number of MA lags is not fund dependent. Specifically, letting H_j and L_j represent the number of MA lags with non-zero weight for fund j, we have $H = max(H_j)$ and $L = max(L_j)$ with $\phi_j^{(h)} = 0$ for any $h > H_j$ and $\pi_j^{(h)} = 0$ for any $h > L_j$.

[8]Since $\Sigma_{h=0}^{L}\overline{\pi}^{(h)} = \Sigma_{h=0}^{L}\Sigma_{j=1}^{J} w_j \cdot \pi_j^{(h)} = \Sigma_{j=1}^{J} w_j \cdot (\Sigma_{h=0}^{L}\pi_j^{(h)}) = 1$

where $\psi_j^{(h)} = \pi_j^{(h)} - \overline{\pi}_j^{(h)}$.

Equations 7 and 8 provide a simple way to recover aggregate and fund-level economic returns in an internally consistent way. First, we get aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ where $\overline{\eta}_t$ are residuals of a MA(H) fit to $\overline{R}_t^o$. Second, we obtain fund-level economic excess returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ where $\widetilde{\eta}_{j,t}$ are residuals from a MA(H) fit (with $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ as covariates) to $\widetilde{R}_{j,t}^o$.[9] Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our 3-step unsmoothing process.[10]

Note that if $\pi_j^{(h)} = \phi_j^{(h)}$, which is the main assumption in GLM (our Equation 1), then:

$$\begin{aligned} \widetilde{R}_{j,t}^o &= \widetilde{\mu}_j \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \widetilde{\eta}_{j,t-h} \ - \ \Sigma_{h=0}^H \ (\phi_j^{(h)} - \overline{\phi}^{(h)}) \cdot \overline{\eta}_{t-h} \\ &= \mu_j \ + \ \Sigma_{h=0}^H \ \phi_j^{(h)} \cdot \eta_{j,t-h} \ - \ \overline{R}_t^o \end{aligned} \tag{9}$$

so that our method is equivalent to recovering η_t directly from the residuals of a MA(H) fit to $\widetilde{R}_{j,t}^o + \overline{R}_t^o - \mu_j = R_{j,t}^o - \mu_j$.

Consequently, our 3-step procedure can be seen as a generalization of GLM that allows aggregate and excess economic returns to have different effects on observed fund-level returns ($\pi_j^{(h)} \neq \phi_j^{(h)}$), but recovers precisely the same unsmoothing procedure if the underlying assumption in GLM ($\pi_j^{(h)} = \phi_j^{(h)}$) is imposed.

The last four columns of Tables 2 and 3 show the results from our 3-step unsmoothing process. From Table 2, unsmoothed fund-level returns display autocorrelations comparable to the ones obtained from 1-step unsmoothing. In contrast to 1-step unsmoothing, how-

[9]As in the 1-step method, if the aggregate and fund-level MA processes are estimated by standard statistical packages (which would normalize $\overline{\pi}^{(0)} = 1$ in step 1 and $\theta_j^{(0)} = 1$ in step 2), then we need to divide the coefficients estimated by the package (and multiple the estimated residuals) by $1 + \overline{\pi}^{(1)} + ... + \overline{\pi}^{(L)}$ in the Step 1 and by $1 + \phi_j^{(1)} + ... + \phi_j^{(H)}$ in Step 2. The $\psi_j^{(h)}$ coefficients do not need to be adjusted in step 2.

[10]Step 2 of the 2-step method requires the estimation of a MA(H) with $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ as covariates to account for the influence of aggregate returns into the fund-level unsmoothing process. One way to further simplify the procedure would be to assume $\pi_j^{(h)} = \overline{\pi}^{(h)}$ since this assumption implies $\psi_j^{(h)} = 0$ (i.e., no covariates in the MA process for observed excess returns). In Internet Appendix A, we report results under this more restrictive structure. There is little change in the 3-step method's performance under this extra restriction. However, we do not impose the restriction $\pi_j^{(h)} = \overline{\pi}^{(h)}$ in the main text to allow for the realistic feature that the unsmoothing processes of different funds depend differently on aggregate returns.

ever, Table 3 shows that our 3-step unsmoothing method effectively drives strategy-level autocorrelations to basically zero.[11] As such, the 3-step approach properly unsmoothes the systematic portion of returns, which has important economic consequences explored in the next section.

2 Unsmoothing Hedge Fund Returns

In this section, we unsmooth hedge fund returns to demonstrate that our 3-step unsmoothing technique improves upon traditional unsmoothing in terms of measuring risk and risk-adjusted performance of funds that invest in illiquid assets. Subsection 2.1 explain the empirical details; subsection 2.2 presents the main results after separating funds into liquidity-based groups; and subsection 2.3 reports results by hedge fund strategy to explore the improvement in risk measurement.

2.1 Empirical Details

(a) Hedge Fund Dataset

We combine data from two major commercial hedge fund databases to build our hedge fund dataset. Specifically, we merge the Lipper Trading Advisor Selection System database (hereafter TASS), accessed in June 2018, with the BarclayHedge database, accessed in April 2018, which produces a representative coverage of the hedge fund universe.[12] Both data providers started keeping a so-called graveyard database of funds that stopped reporting their returns only in 1994. Hence, following the literature, we start our analysis of fund risk

[11] Strategy-level returns from the 3-step unsmoothing method are obtained by aggregating $R_{t,j} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$, not by directly using $\overline{\mu} + \overline{\eta}_t$. As such, the results reported in Table 3 are not mechanical and instead reflect the fact that the 3-step unsmoothing method is able to unsmooth the systematic portion of fund-level returns.

[12] Joenväärä et al. (2019) combine and compare five different hedge fund databases that have been used in academic studies. Their analysis shows that the two datasets used in this study (TASS and BarclayHedge) together with Hedge Fund Research, have the most complete data in terms of the number of funds included and the lack of survivorship bias (after 1994). Joenväärä et al. (2019) also find that the average fund performance is similar across the five databases.

and performance in 1995, which avoids issues associated with survivorship bias.

We apply some standard screens before each observation is included in the main sample. We start by excluding observations with stale (for more than one quarter) Assets Under Management (AUM) or that have missing return or AUM. We then restrict the sample to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample.

In order to minimize the impact of small and idiosyncratic funds and mitigate incubation bias, we perform two standard data screens utilized in the literature. First, funds are only included after reaching the $5 million AUM threshold for the first time, and they are not dropped from the sample in case they fall below this threshold after reaching it. Second, after unsmoothing the returns and estimating factor regressions to obtain each fund's risk loadings, we drop the first 12 monthly returns of each fund before calculating average excess returns, Sharpe ratios, and alphas.[13]

After these initial screens, we merge the data from TASS and BarclayHedge and eliminate duplicate fund observations that exist when the same fund reports to both data providers. In order to do so, we start by identifying possible duplicate funds by fuzzy-matching fund names and fund company names across the two data sources. Then, following Joenväärä et al. (2019), we calculate the correlation of returns for each potential duplicate pair and identify it as a duplicate if the correlation is 99% or higher. Finally, for each duplicate pair identified, we keep the one that has the longest series of valid return and AUM data. The final sample starts in January 1995 and ends in December 2016.

Many of our results separate hedge funds based on their strategies. We identify strategies using the "primary strategy" variable reported by TASS and BarclayHedge. We exclude funds

[13] Jorion and Schwarz (2019) find that dropping the first 12 monthly returns is the most common procedure used in the literature to deal with hedge fund incubation bias and backfill bias. However, they argue that this adjustment alone may not be sufficient and propose an algorithm that allows researchers to impute each fund's initial reporting date and thus address the entirety of backfilled returns. To be conservative and to insure comparability with prior literature, for our main results we simply drop the first 12 monthly returns. In untabulated tests, we find that our key findings regarding the usefulness of the 3-step unsmoothing method are similar or strictly stronger if we follow the procedure proposed by Jorion and Schwarz (2019).

whose strategy is classified as "other" or whose primary strategy does not fall into any of the 12 investment styles identified by Joenväärä et al. (2019). There are only a few funds whose strategy is classified as short bias, hence we group them together with long/short funds. Finally, we exclude funds of funds, because these funds often specialize in investing in different fund categories and therefore they can not be considered an homogeneous group. Tables 1 (discussed earlier) provides the final list of strategies used in our analysis we well as the number of funds in each strategy.

(b) Risk Factors

Our analysis of risk and risk-adjusted performance is based on the FH 8-Factor model, which augments the 7-Factor model in Fung and Hsieh (2001) with an emerging market factor. The risk-free rate and trend-following factors are obtained respectively from Kenneth French's and David A. Hsieh's online data libraries.[14] The 3 equity-oriented risk factors are calculated using equity index data from Datastream, and the 2 bond-oriented factors are calculated using data from the Federal Reserve Bank of St. Louis (both equity and bond factors follow the instructions given on David A. Hsieh's webpage).

(c) 3-step Return Unsmoothing

We perform the 1-step MA unsmoothing following a procedure similar to Getmansky, Lo, and Makarov (2004). That is, we use the AIC criterium to choose the number of smoothing lags (from 0 to 3) in the MA process for observed returns ($R^o_{j,t}$), extract estimated residuals ($\eta_{j,t}$), and add the average return back to obtain economic returns, $R_{j,t} = \mu_j + \eta_{j,t}$.[15] The MA process is estimated using maximum likelihood under $\eta_{j,t} \overset{iid}{\sim} N(0, \sigma^2_{\eta,j})$, as described in GLM.

[14] https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
https://faculty.fuqua.duke.edu/~dah7/HFRFData.htm

[15] It is common in the literature to fix $H = 2$. We allow for H from 0 to 3 to account for heterogeneity across funds, but Internet Appendix A reports results (similar to our baseline analysis) in which we fix $H = 2$.

We follow an analogous procedure for our 3-step MA unsmoothing. First, we take the average return of all funds in a given strategy each month to obtain strategy indexes and perform GLM unsmoothing (as described in the previous paragraph) for each strategy index separately to recover unsmoothed strategy-level returns.[16] Second, we obtain unsmoothed excess returns from Equation 8 (an MA process for excess observed returns with aggregate unsmoothed returns as covarites) also relying on AIC to decide how many MA lags to include.[17] Third, we sum the unsmoothed strategy returns with each fund unsmoothed excess return to obtain fund-level economic returns.

2.2 Results by Liquidity Group

This subsection demonstrates that our 3-step unsmoothing method improves the measurement of risk-adjusted performance relative to traditional (or 1-step) unsmoothing. Since unsmoothing methods are designed to affect only the returns of illiquid funds (i.e., funds with significant return autocorrelation), our analysis classifies funds in groups based on liquidity. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies).[18] We then measure fund-level information within each group and report averages.

The basic problem of smoothed returns is that they understate risk (Getmansky, Lo, and

[16] We rely on equal-weights (instead of value-weights) to construct the strategy indexes because this approach is more consistent with our derivations of the 3-step unsmoothing method (which relies on time-invariant weights). However, Internet Appendix A provides results (similar to our baseline analysis) using value-weights to construct strategy indexes.

[17] For the covariates (aggregate unsmoothed returns), we keep the number of lags fixed to match the number of lags selected during the MA estimation for aggregate returns. This approach is consistent with the derivation of the 3-Step method.

[18] The liquidity ranking obtained from this approach (displayed in Table 1) is consistent with economic logic. The most illiquid strategy is the relative value strategy, which contains funds that attempt to profit from mispricing across securities. If capital markets work well, hedge funds are unlikely to find mispricing opportunities among the pool of liquid securities, and thus tend to invest in relatively illiquid assets. At the other extreme, CTAs represent the most liquid hedge funds as their underlying strategies tend to be based on trend-following and are usually executed using futures contracts, which are marked to market daily.

Makarov (2004)), even though they do not affect average (risk unadjusted) performance. As such, unsmoothing methods are designed to increase return volatility without affecting average returns.

Figure 2(a) shows, for the three strategy liquidity groups, the average (annualized) volatility based on (i) reported returns; (ii) 1-step unsmoothed returns; and (iii) 3-step unsmoothed returns.[19] For the low and mid liquidity strategies, average volatility strongly increases after unsmoothing. For instance, the average fund volatility in the low liquidity strategies increases by 28.8% (from 9.0% to 11.6%) as we unsmooth returns. In contrast, there is almost no change in average volatility as we unsmooth returns of funds in the high liquidity strategies, which shows that unsmoothing methods work well as they should not strongly affect the returns of funds that invest in liquid assets.

Comparing the 3-step unsmoothing with 1-step unsmoothing, we see little increase in average volatility. For instance, after the 3-step unsmoothing, the average volatility of funds in the low liquidity strategies increases by only 1.1% (from 11.6% to 11.7%). It is not surprising that the 3-step unsmoothing method has a relatively small effect on fund volatilities beyond 1-step unsmoothing. The 3-step approach is designed to better unsmooth the systematic portion of returns, not to increase the "unsmoothing strength" of the procedure relative to 1-step unsmoothing. As such, the risk measurement improvement the 3-step method provides (detailed below) is not due to an increase in return volatility.

Figure 2(b) gauges the implications of the volatility increase after return unsmoothing to (annualized) Sharpe ratios.[20] It is clear that Sharpe ratios decline for the low and mid liquidity groups, with the effect being particularly pronouced for the low liquidity group. As with volatility, the 3-step method provides no improvement over the 1-step method in terms

[19]We annualize volatilities by multiplying them by $\sqrt{12}$ (which, strictly speaking, is the correct annualization factor only for returns that are not autocorrelated). Multiplying by this fixed constant does not affect any of the relative comparisons between smoothing methods, but it helps to make the volatility magnitudes more comparable to how investors measure volatility in other asset classes.

[20]For Sharpe ratios, we report cross-fund medians as oppose to averages since the Sharpe ratios of funds with negative average excess returns increase as volatility increases. Nevertheless, results based on cross-fund average Sharpe ratios are similar.

of Sharpe ratios.

Figure 2(c) explores systematic risk by focusing on average R^2s based on the FH 8-Factor model (see Fung and Hsieh (2001)), which effectively captures how much of fund-level return variability is explained by the risk factors most commonly used in the hedge fund literature. Some interesting patterns emerge.

First, 1-step unsmoothing has basically no effect on R^2s (if anything, R^2s decrease), which indicates that even though 1-step unsmoothing increases volatility relative to reported returns, it does not increase the fraction of volatility explained by standard risk factors. In stark contrast, 3-step unsmoothing substantially increases R^2s for funds in the low and mid liquidity strategies. For instance, after 3-step unsmoothing, the average R^2 of funds in the low liquidity strategies increases by 18.8% (from 35.1% to 41.7%) relative to reported returns and by 25.5% (from 33.2% to 41.7%) relative to 1-step unsmoothing.

The R^2 patterns suggest that the 3-step unsmoothing method allows us to better uncover the true systematic risk exposure of hedge funds, which is usually partially concealed because observed returns are smoothed. Given this R^2 result, we should be able to better measure risk-adjusted performance using our 3-step unsmoothing method. Figure 2(d) explores this issue by reporting average (annualized) αs for the three liquidity groups. The 3-step unsmoothing strongly decreases αs for the low and mid liquidity strategies relative to the αs obtained with observed returns or after 1-step unsmoothing. For both groups, average αs decrease by close to 1 percentage point (from 4.1% to 3.0% for the low liquidity group and from 1.9% to 1.0% for the mid liquidity group) relative to observed returns, which is about double the improvement provided by the 1-step unsmoothing method.

Figures 2(e) and 2(f) gauge how unsmoothing affects the statistical significance of fund-level αs. Overall, there is a strong decline in average t^{α}_{stat} as well as on the percentage of significant αs for the low liquidity group. The mid liquidity group still displays an effect, but much weaker since αs are not (on average) very significant in the first place.

Overall, the results indicate that volatility strongly increases after unsmoothing and the fraction of volatility due to systematic risk increases only if we estimate economic returns using our 3-step unsmoothing process. Moreover, unsmoothing returns decreases α, and this effect is stronger if we rely on our 3-step unsmoothing method. Finally, all of these results are present only when studying relatively illiquid funds.

2.3 Results by Hedge Fund Strategy

The previous results suggest that our 3-step unsmoothing method improves upon 1-step unsmoothing in measuring risk-adjusted performance. To better understand what risks (betas) are better measured, it is useful to analyze groups of funds that engage in similar activity, and thus are exposed to similar risks. As such, this subsection reports results by hedge fund strategy.

(a) Volatilities, Sharpe Ratios, R^2s, and Alphas

Figure 3 shows several average statistics by hedge fund strategy. In our description, we refer to "illiquid strategies" as the strategies with autocorrelation coefficient higher than 0.20 (these are the first seven strategies as per Table 3).

Figure 3(a) demonstrates that, for each of the illiquid strategies, volatility increases after unsmoothing, but that our 3-step unsmoothing has little effect beyond 1-step unsmoothing. Figure 3(b) shows how the volatility increase affects (annualized) Sharpe Ratios, which makes it clear that the effect is much larger for the more illiquid strategies. Figure 3(c) plots R^2s relative to the FH 8-Factor model and show that the results observed in the low and mid liquidity groups are present for each of the illiquid strategies separately (with basically no effect on liquid strategies). That is, R^2s do not increase as we 1-step unsmooth returns (if anything, they decrease), but they strongly increase after our 3-step unsmoothing. Figure 3(d) shows that the average α declines in every illiquid strategy and Figures 3(e) and 3(f) make it clear that the statistical decline (i.e., decile in average t^{α}_{stat} and in the percentage of funds with significant α) is much larger for the more illiquid strategies.

Figure 4 reports changes in the main statistics in Figure 3 to focus on the (economic and statistical) improvement from (i) moving from observed returns to 1-step unsmoothed returns and (ii) moving from 1-step unsmoothed returns to 3-step unsmoothed returns. The t_{stat} for each average change is provided on the top of the respective bar. Figures 3(a), 3(b), and 3(c) reinforce the inference obtained from Figure 3 and add that the changes tend to be statistically significant. Figure 3(c) also emphasizes that 1-step unsmoothing significantly improves the measurement of risk-adjusted performance (i.e., decreases αs) relative to reported returns, but the improvement obtained from moving from 1-step unsmoothing to 3-step unsmoothing is even larger than the improvement obtained by decision to unsmooth returns. This result suggests that the economic importance of moving from 1-step to 3-step unsmoothing is at least as high as (and probably even higher) than the importance of unsmoothing returns in the first place.

Overall, the results observed after separating funds based on liquidity groups are largely present for individual strategies as well.

(b) Risk Exposures

The previous results indicate that the FH 8-Factor model explains a significantly higher fraction of the volatility of hedge funds than suggested by looking at observed returns (or at 1-step unsmoothed returns). Below, we ask how much each risk factor contributes to the improvement.

In a factor model with two risk factors, $R_t = \alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t} + \epsilon_t$, R^2 can be decomposed as (the decomposition is analogous for an arbitrary number of risk factors):

$$
\begin{aligned}
R^2 &= Var(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t})/Var(R_t) \\
&= Cov(\alpha + \beta_1 \cdot f_{1,t} + \beta_2 \cdot f_{2,t}, R_t)/Var(R_t) \\
&= \underbrace{\beta_1 \cdot \frac{Cov(f_{1,t}, R_t)}{Var(R_t)}}_{R_1^2} + \underbrace{\beta_2 \cdot \frac{Cov(f_{2,t}, R_t)}{Var(R_t)}}_{R_2^2} \qquad (10)
\end{aligned}
$$

where the second equality follows from the projection orthogonality condition, $Cov(f_{1,t}, \epsilon_t) = Cov(f_{2,t}, \epsilon_t) = 0$, and R_i^2 represents the R^2 portion due to risk factor i.

Figure 5 reports, for each hedge fund strategy, the average R^2 due to each of the eight risk factors in the FH 8-Factor model. For all illiquid strategies (except for the emerging market strategy), the results indicate that the importance of market risk and emerging market risk increase substantially after 3-step unsmoothing. For instance, market risk accounts for almost 18% of the volatility of Event Driven funds after 3-step unsmoothing (an increase of 71.2% relative to the importance of market risk when we look at observed returns). For Emerging Market funds, the only risk factor that displays a substantial increase after 3-step unsmoothing is the emerging market risk factor itself, which is consistent with economic logic. For liquid funds, there is basically no change in the importance of different risk factors after unsmoothing returns.

Also interestingly, the importance of the credit spread risk factor decreases for all illiquid strategies. Since the credit spread risk factor itself is based on somewhat illiquid assets, this result indicates that a portion of the βs associated with the credit spread risk factor is spurious and simply reflect the fact that the credit spread risk factor captures some lagged market information.

Overall, the results indicate that most of the improvement coming from the 3-step unsmoothing method steams from better measuring exposures to market risk and emerging market risk in the underlying illiquid assets held by hedge funds.

3 Unsmoothing Returns of Highly Illiquid Assets

The previous two sections introduce our 3-step unsmoothing method and demonstrate that it substantially improves upon 1-step unsmoothing in the context of hedge funds. While unsmoothing hedge fund returns is a natural application of our 3-step unsmoothing process given the extensive hedge fund literature, unsmoothing is even more important for highly illiquid assets. As such, this section demonstrates how our technique can be used to extract economic returns of highly illiquid assets (or funds that hold such assets). Subsection 3.1 outlines the traditional way to unsmooth returns of highly illiquid assets (see Geltner (1993)) and extends our 3-step process to improve upon it; and Subsection 3.2 applies our 3-step method to unsmooth returns of commercial real estate funds, which are highly illiquid given the appraisal nature of real estate valuation.

3.1 Autoregressive Return Unsmoothing Framework

The baseline unsmoothing framework for highly illiquid assets is due to Geltner (1993) and often referred to in the literature as AR(1) unsmoothing since it implies observed returns follow an autoregressive process of order one.

Geltner (1993) assumes the observed return of fund j at time t is given by (see original paper for the economic motivation):

$$R^o_{j,t} = (1-\theta_j)\cdot R_{j,t} \;+\; \theta_j \cdot R^o_{j,t-1} \tag{11}$$

$$= \mu_j \;+\; \theta_j \cdot (R^o_{t-1} - \mu_j) \;+\; (1-\theta_j)\cdot \eta_{j,t} \tag{12}$$

where θ_j captures the level of "staleness" in observed returns and the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ with $\eta_{j,t} \sim IID$.[21].

The first equality represents the economic assumption that prices are only partially updated so that the observed fund return reflects in part the economic return over the same period and in part the return previously observed. The second equality is an econometric implication that says that, under the given assumption, the observed fund returns follow an AR(1) process

Given Equation 12, we can recover economic returns by estimating an AR(1) process for $R^o_{j,t}$, extracting the estimated residuals ($\epsilon_{j,t} = (1-\theta_j)\cdot\eta_{j,t}$), and using them to get economic returns, $R_{j,t} = \mu_j + \epsilon_{j,t}/(1-\theta_j)$. There are many methods to estimate AR(1) processes, with Ordinary Least Squares (OLS) being the simplest consistent estimator, and thus the method we rely on. This procedure is used by several papers in the literature to unsmooth returns on real estate assets and funds (e.g., Fisher, Geltner, and Webb (1994), Barkham and Geltner (1995), Corgel et al. (1999), Pagliari Jr, Scherer, and Monopoli (2005), Rehring (2012), and

[21]Note also that, under invertibility, Equation 12 implies an MA(∞) representation with weights $\theta^{(h)}_j = \phi^h_j \cdot (1-\phi_j)$, which satisfies $\Sigma^{\infty}_{h=0}\theta^{(h)}_j = 1$.

Pagliari Jr (2017)).

However, as we empirically demonstrate in the next subsection, this AR(1) unsmoothing method faces the same aggregation issue observed with the MA(H) unsmoothing. As such, analogously to the MA(H) case, we generalize the assumption in Equation 11 so that aggregate economic returns are directly included in the return smoothing process. Specifically, we assume:[22]

$$R^o_{j,t} = (1-\phi_j)\cdot \widetilde{R}_{j,t} \ + \ \phi_j \cdot \widetilde{R}^o_{j,t-1} \ + \ (1-\pi_j)\cdot \overline{R}_t \ + \ \pi_j \cdot \overline{R}^o_{j,t-1} \tag{13}$$

$$= \mu_j \ + \ \phi_j \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \ + \ \pi_j \cdot (\overline{R}^o_{t-1} - \overline{\mu}) \ + \ (1-\phi_j)\cdot \widetilde{\eta}_{j,t} \ + \ (1-\pi_j)\cdot \overline{\eta}_t$$

$$= \mu_j \ + \ \phi_j \cdot (\widetilde{R}^o_{j,t-1} - \widetilde{\mu}_j) \ + \ \pi_j \cdot (\overline{R}^o_{t-1} - \overline{\mu}) \ + \ \epsilon_{j,t} \tag{14}$$

where the second equality follows from $R_{j,t} = \mu_j + \eta_{t,j}$ and with $\eta_{j,t} \sim IID$ and the third equality defines $\epsilon_{j,t} = (1-\phi_j)\cdot \widetilde{\eta}_{j,t} \ + \ (1-\pi_j)\cdot \overline{\eta}_t$.

Since the covariates in Equation 14 are observable (in constrast to the MA(H) process), we can directly estimate Equation 14 (by OLS) and obtain estimated coefficients, ϕ_j and π_j, as well as estimated residuals, $\epsilon_{j,t}$. The challenge is that $\epsilon_{j,t}$ reflects both $\widetilde{\eta}_{j,t}$ and $\overline{\eta}_t$. We rely on an aggregation step to separate the two components. Specifically, aggregating $\epsilon_{j,t}$ yields:

$$\overline{\epsilon}_t \ = (1-\overline{\pi})\cdot \overline{\eta}_t \ + \ -\widehat{Cov}(\phi_j, \widetilde{\eta}_{j,t})$$

$$\approx (1-\overline{\pi})\cdot \overline{\eta}_t \tag{15}$$

Similar to the MA(H) case, the structure exposed provides a simple way to recover aggregate and fund-level economic returns in an internally consistent way given the estimates for ϕ_j, π_j, $\epsilon_{j,t}$ obtained from 14. First, we get aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ where $\overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$ with $\overline{\epsilon}_t = \Sigma_{j=1}^J w_j \cdot \epsilon_{j,t}$. Second, we obtain fund-level economic excess returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ where $\widetilde{\eta}_{j,t} = (\epsilon_{j,t} - (1-\pi_j)\cdot \overline{\eta}_t)/(1-\phi_j)$. Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$. This procedure summarizes our AR 3-step unsmoothing method.

[22] This smoothing process reduces to Equation 12 (the smoothing process in Geltner (1993)) if we set $\pi_j = \phi_j = \theta_j$.

Finally, imposing the extra restriction $\pi_j = \phi_j$, which is the main assumption in Geltner (1993), yields:

$$R^o_{j,t} = \mu_j \ + \ \phi_j \cdot (R^o_{j,t-1} - \mu_j) \ + \ (1 - \phi_j) \cdot \eta_{j,t} \tag{16}$$

so that our method is equivalent to recovering η_t directly from the residuals of an AR(1) fit to $\widetilde{R}^o_{j,t} + \overline{R}^o_t = R^o_{j,t}$.

Consequently, our 3-step AR unsmoothing procedure can be seen as a generalization of Geltner (1993) that allows aggregate and excess economic returns to have different effects on observed fund-level returns ($\pi_j \neq \phi_j$), but recovers precisely the same unsmoothing procedure if the underlying assumption in Geltner (1993) ($\pi_j = \phi_j$) is imposed.

In our empirical analysis of real estate funds, we unsmooth log observed returns instead of regular observed returns and then transform the unsmoothed log return into unsmoothed regular returns. This added step assures the unsmoothed regular returns are always above -100%, which is not the case during the 2008 financial crises if we directly unsmooth observed regular returns. However, the overall results are similar either way as demonstrated in the Internet Appendix.

3.2 Unsmoothing Returns of Commercial Real Estate Funds

(a) Commercial Real Estate Dataset

Commercial real estate covers all real estate product types other than single-family homes and is the primary way institutional investors invest in real estate. While commercial real estate has historically been a significant sector in the overall economy, its importance as an investment class has grown dramatically over the last 35 years. The average target allocation for institutional investors has grown from around 2% in the early 1980s to between 10% and 12% in 2018 (PREA (2018)).

There are a number of ways institutional investors can invest in commercial real estate - direct investments, separate accounts, joint ventures, club deals, commingled funds, and publicly traded REITs. Our analysis focuses on US Open-ended Private Real Estate (OPRE) funds, which invest in private capital and are a subset of commingled funds. Our OPRE dataset comes from the National Council of Real Estate Investment Fiduciaries (NCREIF), which is the leading collector of institutional real estate investment information for properties within the US. Our sample includes all OPRE funds that report return data to NCREIF and have at least 36 quarterly observations[23]. The sample consists of 2,000 fund-quarter observations over 96 quarters for a total of 29 funds. There is a minimum of 13 funds in each period. The sample period covers from 1994 through 2017 with the starting data selected to be consistent with the hedge fund analysis (and also because before 1994 the number of funds available is much lower). As of the fourth quarter of 2017, the sample comprises 24 funds with approximately $250 billion in assets under management. Unlike hedge fund data, these data do not suffer from incubation bias, backfill bias, or survivorship bias.

OPRE funds are similar to mutual funds in the sense that they are open to issuing and redeeming shares on a regular basis (quarterly) at stated Net Asset Values (NAVs). However, fund NAVs are based on the cumulative appraised values of the individual assets they hold, and thus NAV-based (i.e., observed) returns reflect (highly) smoothed returns. Therefore, OPRE funds provide a natural asset class to explore the effects of our 3-step AR unsmoothing method.

There is no consensus on the appropriate factor model to measure risk exposures of real estate funds. As such, our analysis relies on a simple model that includes as risk factors excess returns on the equity and real estate (public) markets.[24] Despite its simplicity, we show that this factor model drives the average α of OPRE funds to (roughly) zero after 3-step unsmoothing.

[23]We require at least 36 observations to be consistent with the Hedge Fund analysis and to assure the smoothing process is estimated with some precision. However, including all OPRE funds available in the dataset yields similar results to the ones we report.

[24]For excess returns on the equity market, we use the market risk factor in Kenneth French's data library (https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). For excess returns on the real estate market, we use the difference between quarterly returns on the FTSE NAREIT All Equity REIT Index (which captures the publicly traded REIT market) and the one-month Treasury bill rate compounded over each quarter.

(b) Autocorrelations

Table 4 provides average autocorrelations for OPRE fund returns as well as autocorrelations for the aggregate (equal-weighted average) of all OPRE fund returns. Confirming the intuition that the appraisal nature of real estate valuation induces (highly) smoothed returns, we find that the average 1-quarter autocorrelation of observed returns is 0.72, with every fund displaying a statistically significant autocorrelation. In fact, returns are so persistent that the average autocorrelation is still 0.18 at 4-quarters (and statistically significant for 41.4% of the funds). Returns are even more autocorrelated at the aggregate level, with the aggregate OPRE returns displaying a 1-quarter autocorrelation of 0.88 (with p-value=0.0%) and a 4-quarter autocorrelation of 0.20 (with p-value=5.5%).

Applying the 1-step AR unsmoothing to OPRE funds, we have that autocorrelations at the fund level mostly disappear. However, the 1-quarter autocorrelation for the aggregate series remains extremely strong (at 0.54 with p-value=0.0%) and even the 2-quarter autocorrelation remains relevant (at 0.20 with p-value=5.6%). This result indicates the 1-step AR unsmoothing does not fully unsmooth the systematic portion of returns.

One possibility is that fund returns follow a general AR(H) process as opposed to an AR(1) process. To check this possibility, we report partial autocorrelations (i.e., coefficients from a multivariate regression that includes lagged returns from up to 4 quarters). It is clear that fund-level observed returns are well captured using an AR(1) since only the 1-quarter partial autocorrelation is relevant (the average is 0.76 and it is significant for 96.6% of the funds). For aggregate returns, however, the 2nd order partial autocorrelation is also important, which has non-trivial consequences (detailed below) for the performance of the 3-step method.

After applying the 3-step AR unsmoothing method, the aggregate series is less autocorrelated with 1-quarter autocorrelation of 0.45 (p-value=0.0%) and the 2-quarter autocorrelation of 0.16 (p-value=12.6%). The aggregate series remains autocorrelated, however, so the method improves upon 1-step unsmoothing, but is not able to fully unsmooth the systematic component of returns. The 1-quarter fund-level autocorrelation is also not fully driven to zero (the average is 0.39 with 89.7% of the funds still displaying significant autocorrelation). Despite this limitation, we show below that the 3-step method performs well in terms of increasing risk exposures and decreasing risk adjusted performance. As such, the 3-step AR unsmoothing provides a substantial improvement from the perspective of performance evaluation despite not being 100% effective in driving autocorrelation to zero.

(c) Performance of 1-step and 3-step Unsmoothing

The upper panel of Table 5 reports basic fund statistics based on observed return, 1-step unsmoothed returns, and 3-step unsmoothed returns. Annualized expected returns are 5.5% and, by construction, do not change as we unsmooth returns, while (annualized) volatility starts at 7.9% for observed returns, increases to 21.8% after 1-step unsmoothing, and increases even further to 27.7% after 3-step unsmoothing. Interestingly, in the case of OPRE funds, R^2 increases (from 4.0% to 14.8%) as we 1-step unsmooth returns. R^2 increases even further (from 14.8% to 20.8%) as we move from 1-step to 3-step unsmoothing. These results indicate that unsmoothing has the potential to largely affect risk measurement and, consequently, estimated risk-adjusted performance.

When analysing observed returns, OPRE funds seem to provide a substantial average α of 4.9% per year (with 82.8% of the funds displaying statistically significant α). This result is a consequence of the extremely low average exposure to equity ($\beta_e = 0.03$) and real estate ($\beta_{re} = 0.04$) public markets. After 1-step unsmoothing returns, the average exposures to the equity ($\beta_e = 0.05$) and real estate ($\beta_{re} = 0.30$) markets increase, driving the average α down to 2.1%, with only 10.3% of the funds displaying statistically significant α. Average risk exposures increase even further after 3-step unsmoothing ($\beta_e = 0.05$ and $\beta_{re} = 0.56$) so that the average α becomes -0.5% and statistically insignificant (or significantly negative) for all funds.

The lower panel of Table 5 reports the same results as the upper panel, but focuses on how βs and α change as we unsmooth returns. The key message is that the increase in βs and decline in α obtained by 1-step unsmoothing returns is about the same as the improvement obtained when moving from 1-step to 3-step unsmoothing. The t_{stat} values in brackets also show that the average changes are highly significant from a statistical perspective.

Overall, the results indicate that, despite still leaving some autocorrelation in aggregate returns, the 3-step AR unsmoothing method provides a substantial improvement over 1-step AR unsmoothing in terms of measuring risk exposure and risk-adjusted performance of commercial real estate funds. The economic gains obtained by moving from 1-step to 3-step unsmoothing are roughly similar to the gains of unsmoothing in the first place.

4 Conclusion

In this paper, we find that traditional return unsmoothing methods used to recover economic return estimates from observed returns of illiquid assets do not fully unsmooth the systematic portion of the returns, and thus understate systematic risk exposures and overstate risk-adjusted performance. To address this issue, we provide an adjustment to traditional return unsmoothing techniques and apply it to hedge funds and commercial real estate funds. In doing so, we find that measurements of risk exposures and risk-adjusted performance substantially improve. The improvement is stronger for more illiquid hedge funds, and the results are even stronger when we apply our new technique to commercial real estate funds, which are highly illiquid due to the appraisal nature of real estate valuation.

Our results demonstrate the economic importance of properly unsmoothing the returns of illiquid assets. They also raise the possibility that some previously estimated alphas of funds that invest in illiquid assets are partially due to mismeasured systematic risk. We provide initial evidence consistent with this argument in the context of hedge funds and commercial real estate funds and leave further explorations in this direction to future research.

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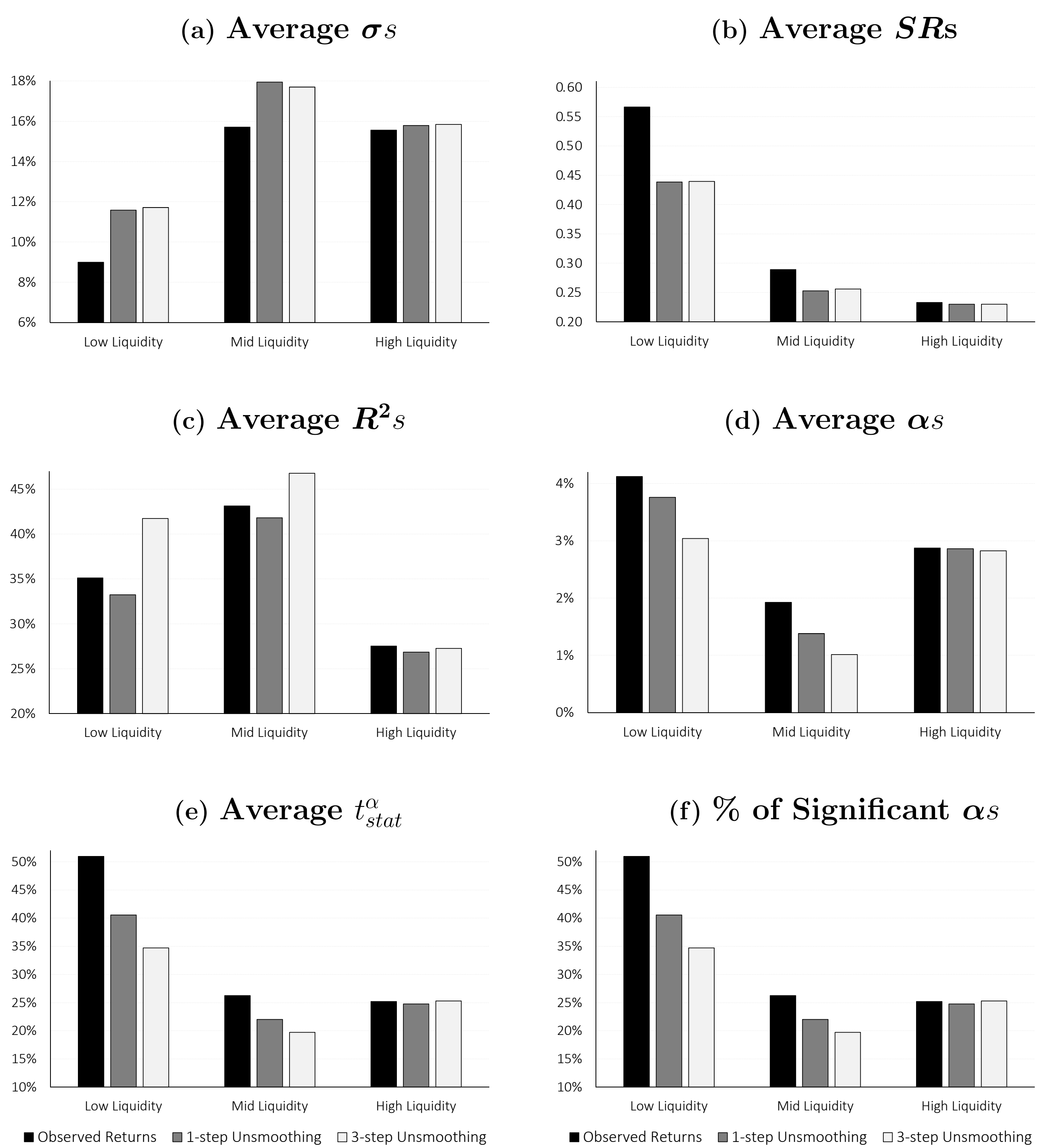


Figure 2

Hedge Fund Risk and Performance by Strategy Liquidity

The figure plots average fund-level information for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2016. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

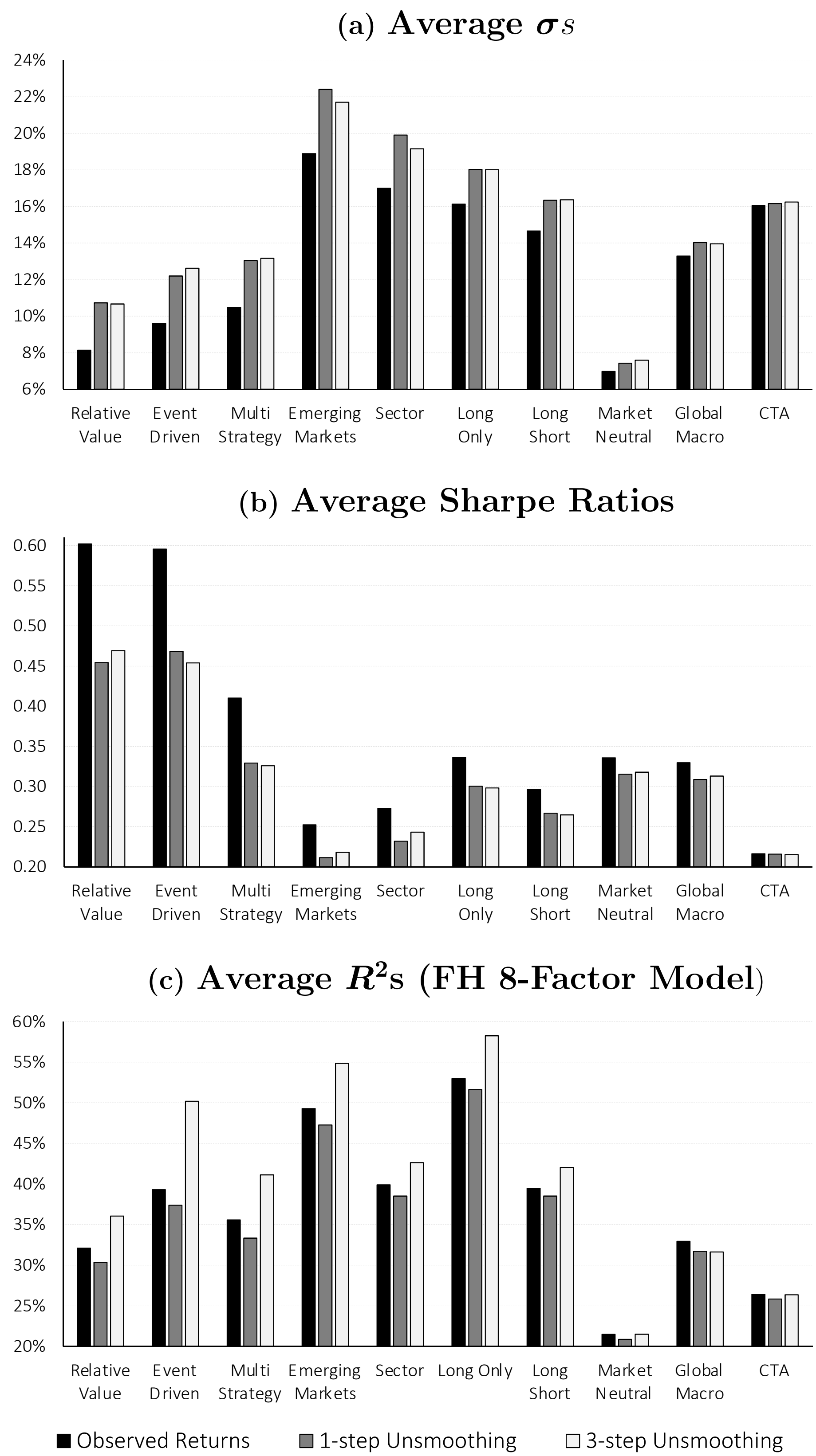
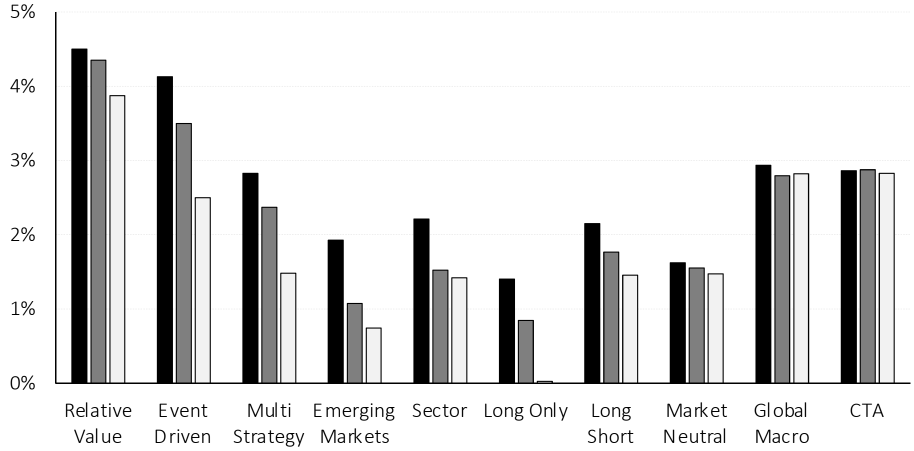


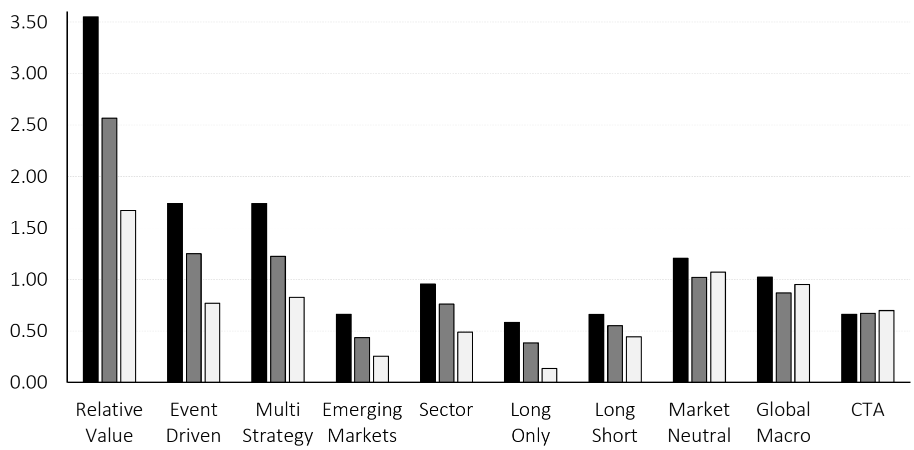
Figure 3
Hedge Fund Risk and Performance by Strategy

The figure plots average fund-level information by hedge fund strategy using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2016. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

(d) **Average αs (FH 8-Factor Model)**



(e) **Average t^{α}_{stat}**



(f) **% of Funds with α Significant at 10%**

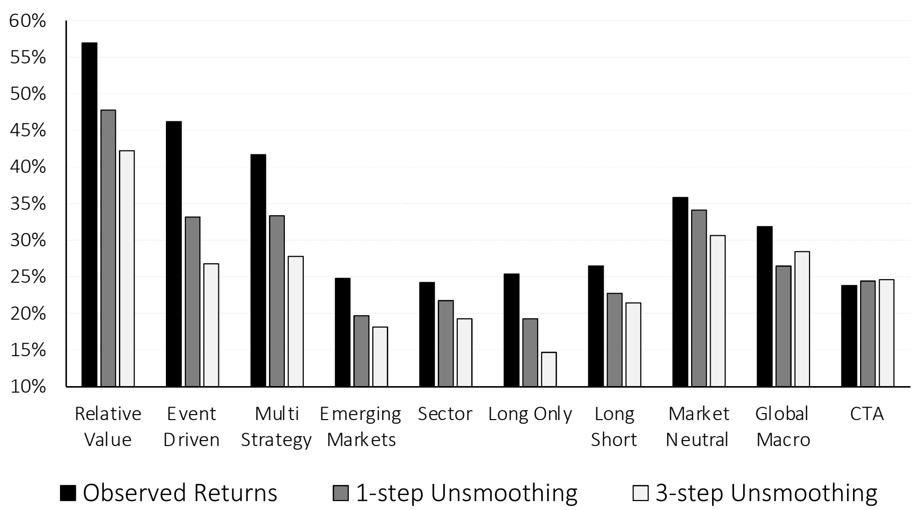


Figure 3 (Cont'd)
Hedge Fund Risk and Performance by Strategy

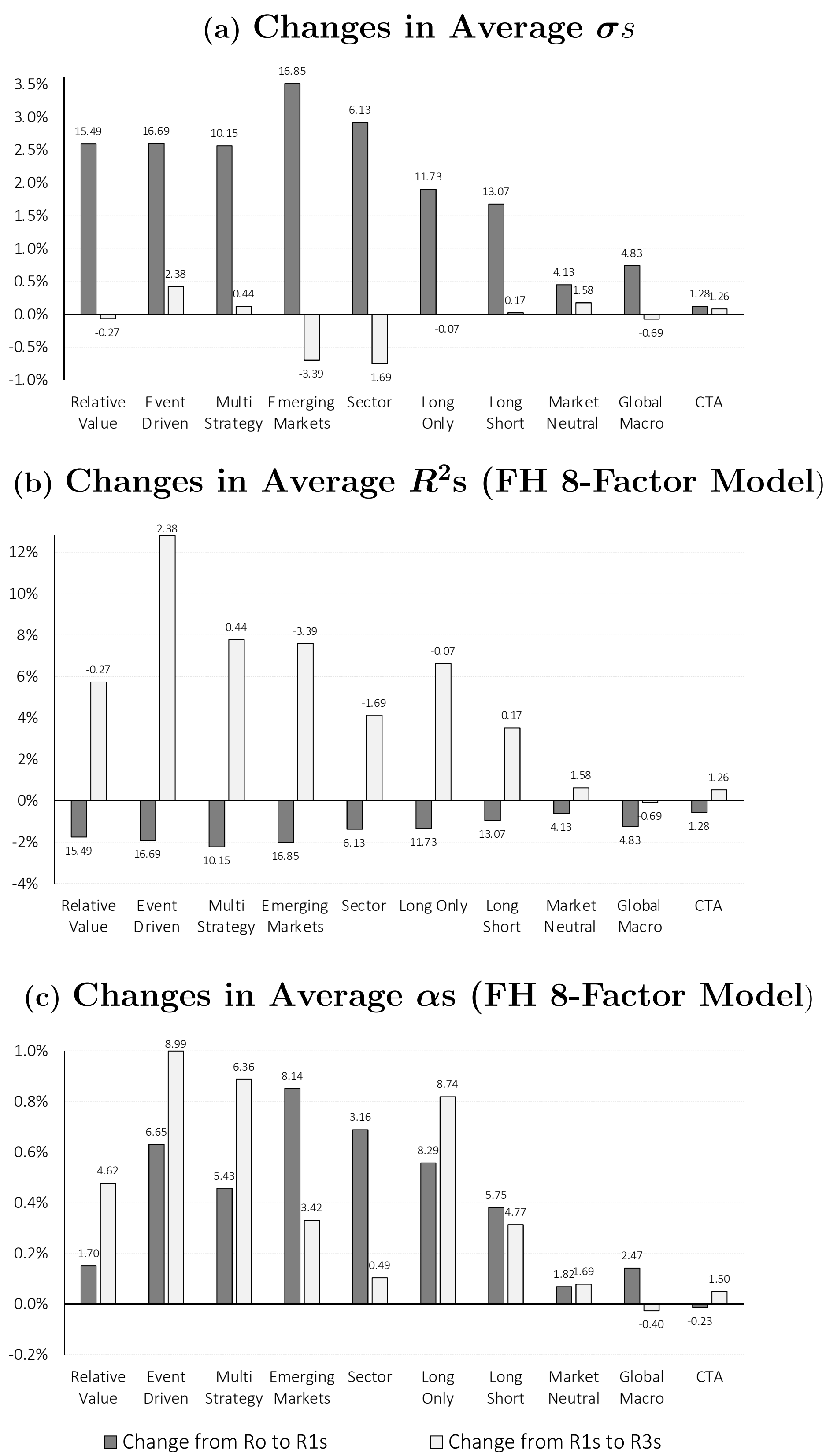


Figure 4
Changes in Hedge Fund Risk and Performance by Strategy

The figure plots increases in σ and R^2 (declines in α) with their t_{stat} by hedge fund strategy as we move (i) from observed to 1-step unsmoothed returns and (ii) from 1-step to 3-step unsmoothed returns. R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2016. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

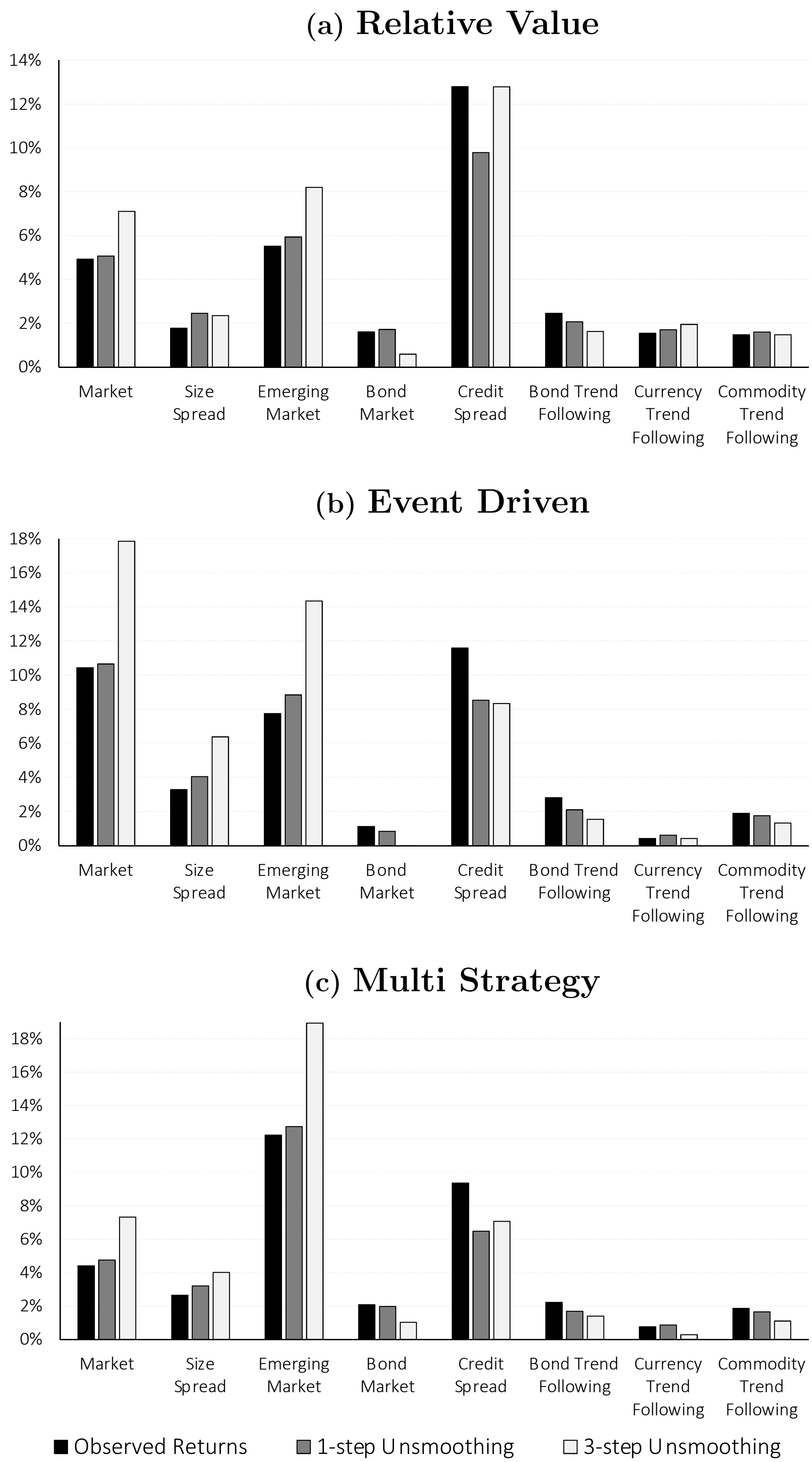
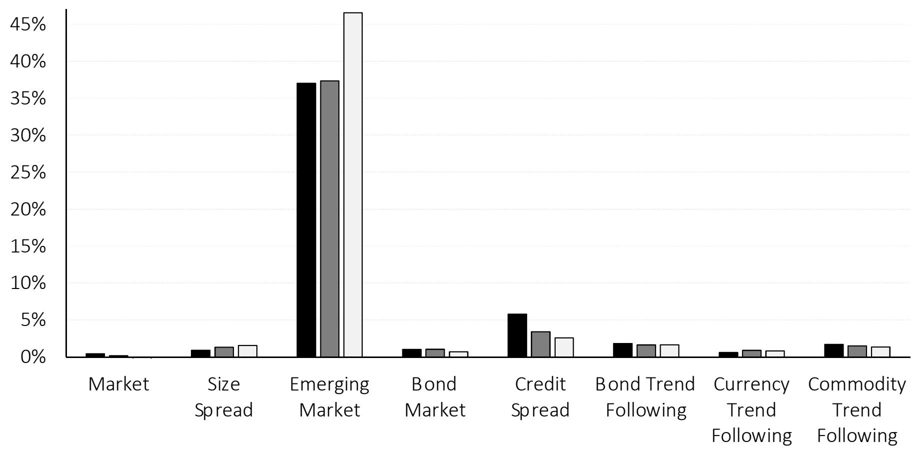


Figure 5

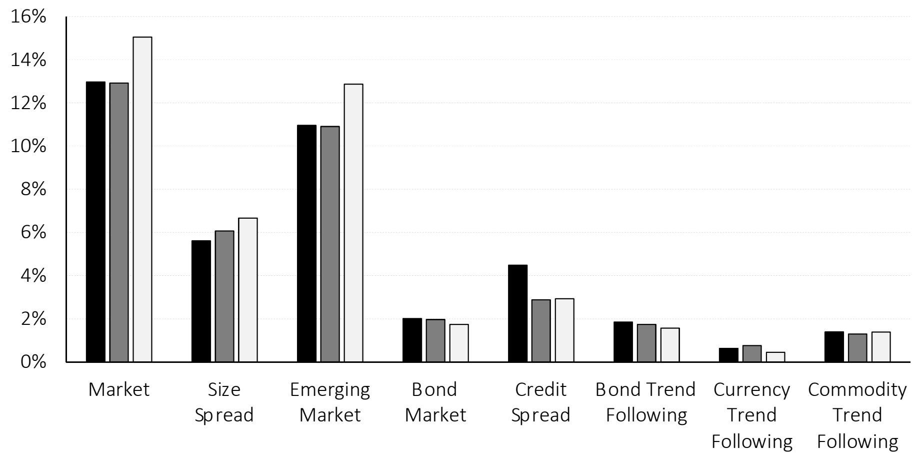
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

The figure plots, for each hedge fund strategy, the average R^2 from factor regressions decomposed into the effect of each risk factor (see Equation 10). We use the risk factors in the FH 8-Factor model that builds on Fung and Hsieh (2001). The sample goes from January 1995 to December 2016. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

(d) Emerging Markets



(e) Sector



(f) Long Only

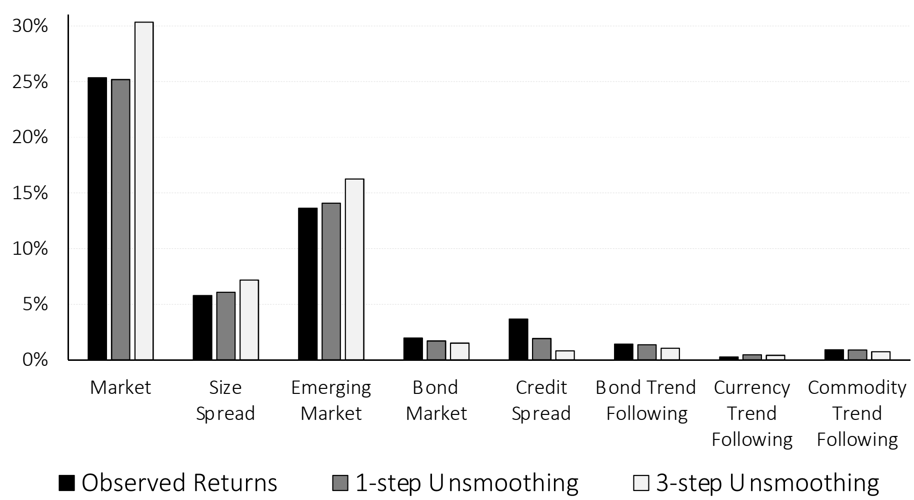
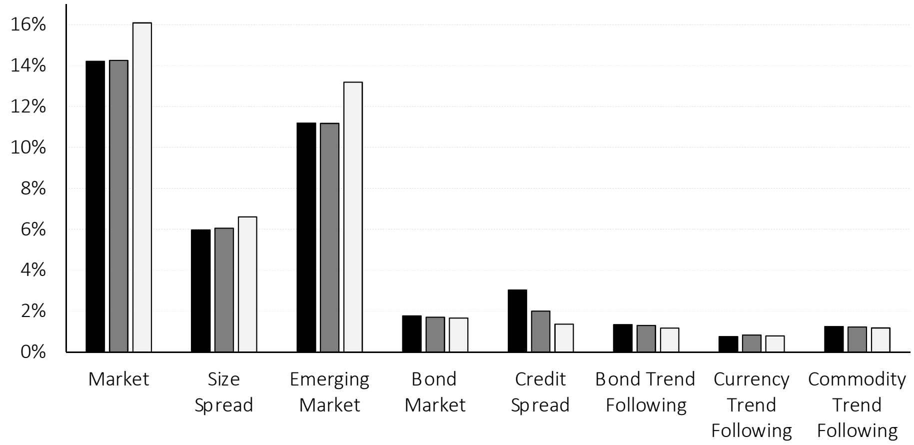
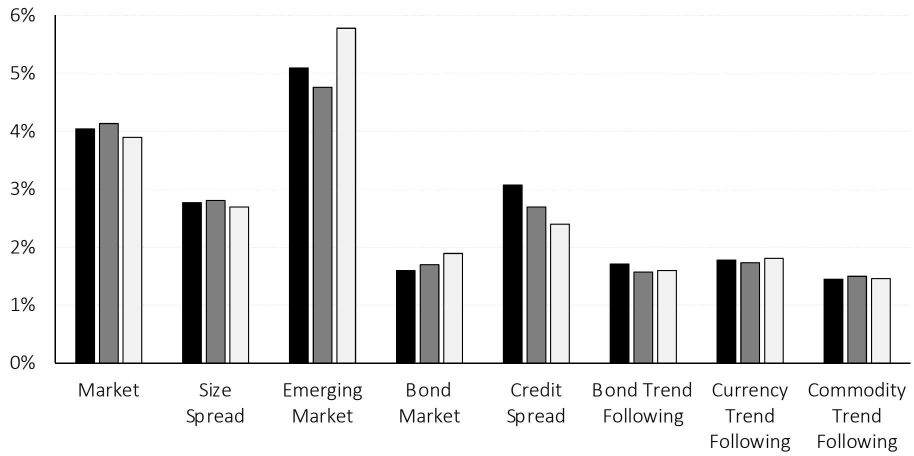


Figure 5 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(g) Long Short



(h) Market Neutral



(i) Global Macro

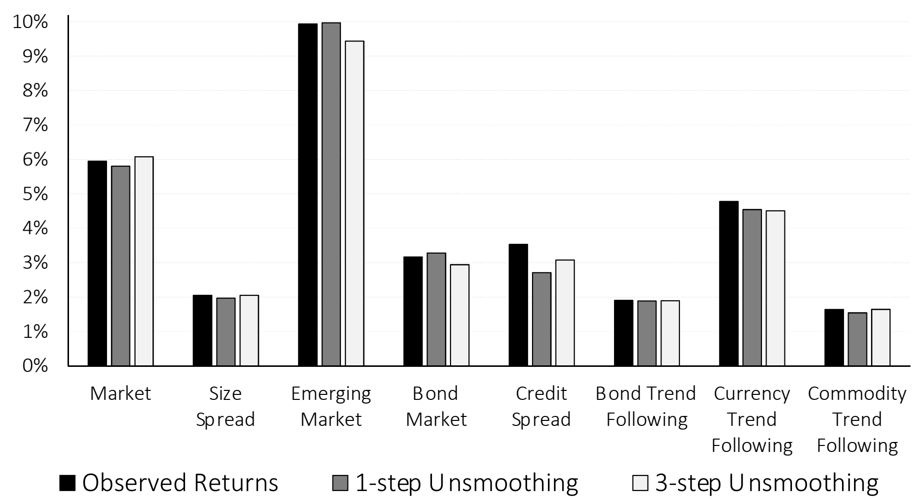


Figure 5 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

(j) **CTA**

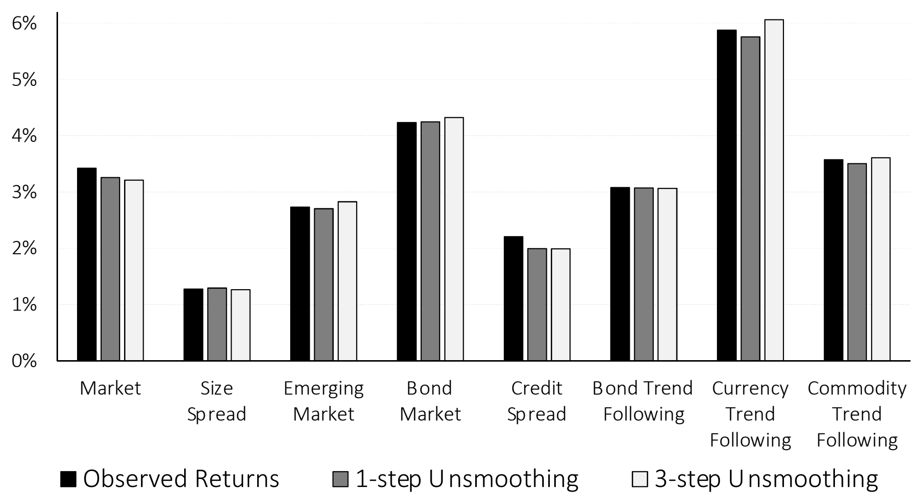


Figure 5 (Cont'd)
Decomposing Hedge Fund R^2s into the Effect of Each Risk Factor

Table 1
Hedge Fund Strategies and Summary Statistics

The table reports the total number of hedge funds (N), the average number of months per hedge fund ($\overline{T}$) and other average fund-level statistics for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient (Cor_1). All statistics are based on observed returns. The sample goes from January 1995 to December 2016 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

Hedge Fund	Sample Size		Fund-level			
Strategies	N	$\overline{T}$	Cor_1	σ	$\mathbb{E}[r]$	$\mathbb{E}[r]/\sigma$
Relative Value	611	81	0.30	8.1%	4.9%	0.60
Event Driven	422	91	0.25	9.6%	5.7%	0.60
Multi Strategy	180	96	0.24	10.5%	4.3%	0.41
Emerging Mkts	585	89	0.17	18.9%	4.8%	0.25
Sector	322	85	0.14	17.0%	4.6%	0.27
Long Only	457	99	0.11	16.1%	5.4%	0.34
Long-Short	906	89	0.10	14.7%	4.3%	0.30
Market Neutral	173	79	0.08	7.0%	2.3%	0.34
Global Macro	204	87	0.07	13.3%	4.4%	0.33
CTA	967	92	0.01	16.0%	3.5%	0.22

Table 2
Autocorrelations of Hedge Fund Returns

The table reports average fund-level autocorrelations (from 1 to 4 months) for hedge fund returns by strategy, with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. The numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. The sample goes from January 1995 to December 2016 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

Hedge Fund	Observed Returns				1-step Unsmoothing				3-step Unsmoothing			
Strategies	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Relative Value	0.30	0.15	0.16	0.11	0.01	0.03	0.07	0.09	0.00	-0.04	0.08	0.06
	(65.1%)	(38.0%)	(34.4%)	(25.5%)	(0.5%)	(3.3%)	(7.2%)	(17.7%)	(11.5%)	(6.5%)	(22.7%)	(13.6%)
Event Driven	0.25	0.13	0.10	0.05	0.02	0.03	0.04	0.03	-0.02	0.00	0.03	0.05
	(60.9%)	(36.3%)	(29.6%)	(17.5%)	(0.2%)	(2.6%)	(5.7%)	(10.7%)	(3.6%)	(2.6%)	(9.0%)	(16.4%)
Multi Strategy	0.24	0.11	0.07	0.03	0.02	0.02	0.03	0.01	-0.01	-0.02	0.00	0.05
	(60.6%)	(36.1%)	(23.3%)	(13.3%)	(0.6%)	(0.6%)	(2.8%)	(7.2%)	(1.1%)	(2.2%)	(3.3%)	(13.9%)
Emerging Mkts	0.17	0.07	0.06	0.02	0.02	0.01	0.02	0.01	0.01	0.06	0.04	0.01
	(44.1%)	(24.6%)	(16.2%)	(7.9%)	(0.7%)	(1.5%)	(4.3%)	(6.7%)	(7.9%)	(18.3%)	(8.2%)	(5.0%)
Sector	0.14	0.04	0.02	0.05	0.02	0.00	0.00	0.03	0.01	0.03	0.00	0.05
	(34.8%)	(17.7%)	(9.0%)	(12.1%)	(1.6%)	(1.6%)	(1.9%)	(8.7%)	(5.3%)	(9.9%)	(4.0%)	(9.6%)
Long Only	0.11	0.03	0.04	0.01	0.01	0.01	0.02	0.01	-0.02	0.03	0.02	0.01
	(34.4%)	(10.5%)	(9.4%)	(8.3%)	(0.0%)	(0.4%)	(3.5%)	(8.1%)	(4.6%)	(8.1%)	(4.8%)	(7.0%)
Long-Short	0.10	0.03	0.03	0.01	0.01	0.00	0.01	0.01	-0.01	0.02	0.01	0.01
	(29.2%)	(13.0%)	(10.5%)	(8.4%)	(0.9%)	(2.1%)	(3.0%)	(7.1%)	(2.2%)	(8.3%)	(6.2%)	(7.3%)
Market Neutral	0.08	0.02	0.00	0.02	0.01	-0.01	-0.01	0.02	-0.01	0.02	-0.03	0.03
	(23.7%)	(15.0%)	(13.9%)	(12.7%)	(2.3%)	(2.3%)	(5.8%)	(8.7%)	(3.5%)	(10.4%)	(5.8%)	(13.9%)
Global Macro	0.07	0.02	-0.02	0.01	0.00	0.01	-0.03	0.00	0.02	0.00	-0.02	0.00
	(21.6%)	(9.8%)	(7.4%)	(7.4%)	(1.5%)	(2.0%)	(4.4%)	(4.4%)	(8.3%)	(2.9%)	(3.9%)	(7.4%)
CTA	0.01	-0.01	-0.03	0.01	-0.01	-0.01	-0.03	0.00	0.00	-0.01	-0.03	0.01
	(10.9%)	(7.3%)	(4.4%)	(7.5%)	(0.7%)	(2.2%)	(1.4%)	(6.5%)	(6.1%)	(5.9%)	(3.5%)	(7.9%)

Table 3
Autocorrelations of Aggregated Hedge Fund Returns

The table reports autocorrelations (from 1 to 4 months) for returns of each hedge fund strategy index (i.e., equal-weighted portfolio of all funds following the given strategy), with strategies sorted based on the 1st order (average fund-level) autocorrelation coefficient. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. The numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. The sample goes from January 1995 to December 2016 and is restricted to US-dollar funds that report net-of-fees returns, have at least 36 uninterrupted monthly observations, and reach $5 million in AUM at some point in the sample, with fund observations included only after reaching the $5 million AUM threshold for the first time. See Section 1 for unsmoothing methods and Section 2.1 for further empirical details.

Hedge Fund	Observed Returns				1-step Unsmoothing				3-step Unsmoothing			
Strategies	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Relative Value	0.51	0.27	0.13	0.10	0.29	0.13	0.04	0.08	0.00	0.06	0.10	0.09
	(0.0%)	(0.0%)	(3.0%)	(10.3%)	(0.0%)	(4.2%)	(52.2%)	(17.7%)	(94.7%)	(35.4%)	(12.0%)	(16.1%)
Event Driven	0.46	0.25	0.18	0.09	0.24	0.13	0.10	0.08	0.00	0.01	0.04	0.09
	(0.0%)	(0.0%)	(0.3%)	(13.6%)	(0.0%)	(4.2%)	(10.8%)	(22.4%)	(97.2%)	(86.4%)	(47.9%)	(13.1%)
Multi Strategy	0.47	0.32	0.23	0.14	0.21	0.17	0.12	0.10	0.02	0.03	0.05	0.13
	(0.0%)	(0.0%)	(0.0%)	(2.3%)	(0.1%)	(0.6%)	(5.3%)	(10.5%)	(69.1%)	(58.5%)	(44.1%)	(3.4%)
Emerging Mkts	0.29	0.10	0.08	0.06	0.16	0.05	0.05	0.05	0.01	0.10	0.05	0.05
	(0.0%)	(10.4%)	(20.4%)	(30.7%)	(0.9%)	(40.5%)	(42.3%)	(38.9%)	(81.5%)	(9.3%)	(45.5%)	(45.2%)
Sector	0.22	0.05	0.03	0.02	0.10	0.01	0.02	0.03	0.01	0.04	0.01	0.03
	(0.0%)	(45.6%)	(61.4%)	(70.0%)	(12.2%)	(84.8%)	(77.5%)	(60.5%)	(92.0%)	(52.3%)	(87.7%)	(59.4%)
Long Only	0.22	0.06	0.04	-0.02	0.12	0.04	0.03	-0.02	0.01	0.06	0.03	-0.02
	(0.0%)	(31.0%)	(53.5%)	(80.1%)	(6.3%)	(51.1%)	(68.7%)	(77.7%)	(85.5%)	(35.2%)	(65.6%)	(76.1%)
Long-Short	0.22	0.10	0.07	0.03	0.11	0.07	0.05	0.03	0.01	0.09	0.05	0.04
	(0.0%)	(11.0%)	(25.2%)	(57.7%)	(6.6%)	(26.2%)	(39.1%)	(62.8%)	(93.2%)	(14.8%)	(39.6%)	(57.3%)
Market Neutral	0.17	0.06	0.08	0.25	0.11	0.01	0.09	0.23	-0.02	0.07	0.04	0.22
	(0.7%)	(32.3%)	(16.5%)	(0.0%)	(6.8%)	(82.4%)	(15.9%)	(0.0%)	(79.2%)	(28.8%)	(50.7%)	(0.0%)
Global Macro	0.05	-0.06	-0.01	0.05	0.01	-0.03	0.01	0.05	0.04	-0.06	-0.01	0.05
	(45.0%)	(33.4%)	(80.7%)	(37.6%)	(92.0%)	(57.2%)	(93.0%)	(37.9%)	(49.8%)	(33.1%)	(86.9%)	(40.1%)
CTA	-0.02	-0.04	-0.02	-0.03	-0.03	-0.02	-0.01	-0.04	-0.02	-0.04	-0.02	-0.03
	(76.7%)	(46.9%)	(72.8%)	(62.8%)	(67.5%)	(72.5%)	(86.9%)	(54.6%)	(74.9%)	(47.4%)	(75.4%)	(58.1%)

Table 4
Autocorrelations of OPRE Fund Returns

The table reports (average fund-level and aggregate) autocorrelations (from 1 to 4 quarters) for US Open-ended Private Real Estate (OPRE) funds. Autocorrelations are based on observed returns, 1-step unsmoothed returns (as in Geltner (1993)), and 3-step unsmoothed returns. In the upper panel, the numbers in parentheses reflect the fraction of funds with the respective autocorrelation being significant at 10% level. In the lower panel, the numbers in parentheses reflect the p-value for the test of whether the respective autocorrelation differs from zero. Partial autocorrelations refer to coefficients from a multivariate regression that includes lagged returns from up to 4 quarters. The sample goes from 1994 through 2017 and is restricted to OPRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Section 3.1 for the AR unsmoothing methods used and Section 3.2 for further empirical details.

		Autocorrelations				**Partial Autocorrelations**			
	Returns	Cor_1	Cor_2	Cor_3	Cor_4	Cor_1	Cor_2	Cor_3	Cor_4
Fund Level	**Observed**	0.72	0.54	0.35	0.18	0.76	0.03	0.03	-0.15
		(100%)	(96.6%)	(93.1%)	(41.4%)	(96.6%)	(17.2%)	(10.3%)	(0.0%)
	1-step	0.02	0.10	0.03	-0.02	0.05	0.08	0.08	-0.07
		(10.3%)	(37.9%)	(3.4%)	(3.4%)	(10.3%)	(34.5%)	(13.8%)	(3.4%)
	3-step	0.39	0.15	0.00	-0.08	0.42	-0.04	0.01	-0.08
		(89.7%)	(17.2%)	(0.0%)	(0.0%)	(89.7%)	(0.0%)	(3.4%)	(0.0%)
Aggregate Level	**Observed**	0.88	0.66	0.43	0.20	1.25	-0.46	0.17	-0.18
		(0.0%)	(0.0%)	(0.0%)	(5.5%)	(0.0%)	(0.8%)	(31.3%)	(8.3%)
	1-step	0.54	0.20	0.08	-0.08	0.62	-0.18	0.15	-0.18
		(0.0%)	(5.6%)	(44.8%)	(42.2%)	(0.0%)	(15.3%)	(23.4%)	(9.4%)
	3-step	0.45	0.16	0.00	-0.08	0.47	-0.03	-0.03	-0.07
		(0.0%)	(12.6%)	(97.4%)	(42.6%)	(0.0%)	(80.0%)	(81.4%)	(52.6%)

Table 5
Risk and Performance of OPRE Funds

The table reports (average fund-level) statistics related to the risk and performance of US Open-ended Private Real Estate (OPRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from 1994 through 2017 and is restricted to OPRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Section 3.1 for the AR unsmoothing methods used and Section 3.2 for further empirical details.

	Returns	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	R^2	α	β_e	β_{re}
Statistic	Observed, R_o	5.5%	7.9%	0.70	4.0%	4.9%	0.03	0.04
						(82.8%)	(0.0%)	(17.2%)
	1-step, R_{1s}	5.5%	21.8%	0.25	14.8%	2.1%	0.05	0.30
						(10.3%)	(0.0%)	(65.5%)
	3-step, R_{3s}	5.5%	27.7%	0.20	20.8%	-0.5%	0.05	0.56
						(0.0%)	(0.0%)	(89.7%)
Change in Statistic	R_o to R_{1s}	0.0%	13.9%	-0.45	10.8%	-2.8%	0.02	0.26
						[-7.94]	[0.57]	[6.68]
	R_{1s} to R_{3s}	0.0%	6.0%	-0.05	6.0%	-2.6%	0.00	0.26
						[-6.88]	[0.01]	[6.32]
	R_o to R_{3s}	0.0%	19.9%	-0.50	16.8%	-5.4%	0.02	0.52
						[-14.94]	[1.50]	[14.79]

Internet Appendix

"Unsmoothing Returns of Illiquid Assets"

By Spencer Couts, Andrei S. Gonçalves, and Andrea Rossi

This Internet Appendix contains further results that supplement the main findings in the paper by performing several robustness checks that modify some of our empirical specifications related to sample construction and econometric design.

A Robustness Analyses

This section performs a series of robustness analyses that modify different aspects of our empirical design. To conserve space, we only report the core results for each alternative specification studied, but other results are also similar to what we report in the text.

A.1 3-step Unsmoothing with Value-weighted Aggregate Returns

While developing our 3-step unsmoothing method, we relied on time-invariant weights, w_j. For this reason, we used equal-weights in our empirical analysis (as opposed to value-weights) when unsmoothing aggregate (or strategy-level) returns. For robustness, we repeat our analysis here after replacing equal-weights with value-weights (i.e., weights based on NAV). All other aspects of the analysis are kept fixed (including the fact that we are report equal-weighted averages of statistics, not value-weighted averages).

Figure IA.1 replicates Figure 2 in the main text after replacing equal-weights with value-weights to construct strategy indexes. It is clear that all results presented in the main text remain valid after relying on value-weighted strategy indexes. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.1 replicates Table 5 in the main text after replacing value-weights with equal-weights to construct the aggregate OPRE fund return. Similar to what we report in the main text, the 3-step method continues to substantially increase risk exposures, with the average exposure of OPRE funds to the public real estate market increasing from 0.04 to 0.49. Consequently, the 3-step method effectively drives the average α of OPRE funds to (roughly) zero.

A.2 3-step Unsmoothing with Common Smoothness Component

While the 3-step unsmoothing method is general enough to allow for the realistic feature that the unsmoothing processes of different funds depend differently on aggregate returns, we can simplify the method further by assuming that the the underlying illiquidity of the asset class induces a common smoothness component in fund-level returns. This assumption translates into $\pi_j^{(h)} = \overline{\pi}^{(h)}$ for hedge funds and $\pi_j = \overline{\pi}$ for real estate funds. Under this extra assumption, the second step of the 3-step MA unsmoothing does not require the $\overline{\eta}_t$, $\overline{\eta}_{t-1}$, ..., $\overline{\eta}_{t-L}$ covariates (since $\psi_j^{(h)} = 0 \ \forall \ j, h$). Moreover, the 3-step AR unsmoothing can be performed with steps analogous to the 3-step AR unsmoothing.[25]

Figure IA.2 replicates Figure 2 in the main text after imposing $\pi_j^{(h)} = \overline{\pi}^{(h)}$. It is clear that all results presented in the main text remain valid after the extra restriction. In particular, the 3-step method has little effect on volatility, but it increases R^2s and decreases αs.

Table IA.2 replicates Table 5 in the main text after replacing value-weights with equal-weights to construct the aggregate OPRE fund return. Similar to what we report in the main text, the 3-step method continues to substantially increase risk exposures, with the average exposure of OPRE funds to the public real estate market increasing from 0.04 to 0.55. Consequently, the 3-step method effectively drives the average α of OPRE funds to (roughly) zero.

A.3 MA 3-step Unsmoothing with $H = 2$

In the literature, it is common to rely on a MA(2) when unsmoothing hedge fund returns. In the main text, we instead use the AIC criterium to choose the number of smoothing lags (between 0, 1, 2, and 3) for the observed return process. This approach allows for heterogeneity across funds. For robustness, we repeat our analysis here after fixing $H = 2$. Figure IA.3 replicates Figure 2 in the main text after fixing $H = 2$ and demonstrates that the results are very similar to what we report in Figure 2.

[25]First, we get aggregate economic returns from $\overline{R}_t = \overline{\mu} + \overline{\eta}_t$ where $\overline{\eta}_t = \overline{\epsilon}_t/(1-\overline{\pi})$ with $\overline{\epsilon}_t$ representing residuals of an AR(1) fit to $\overline{R}_t^o$. Second, we obtain fund-level economic excess returns from $\widetilde{R}_{j,t} = \widetilde{\mu}_j + \widetilde{\eta}_{j,t}$ where $\widetilde{\eta}_{j,t} = \widetilde{\epsilon}_{j,t}/(1-\phi_j)$ with $\widetilde{\epsilon}_{j,t}$ representing residuals of a AR(1) fit to $\widetilde{R}_{j,t}^o$. Third, we recover fund-level economic returns from $R_{t,j} = \overline{R}_t + \widetilde{R}_{j,t} = \mu_j + \overline{\eta}_t + \widetilde{\eta}_{j,t}$.

A.4 AR 3-step Unsmoothing with Regular Returns

In the main text, we apply the 3-step AR unsmoothing to log returns and transform the unsmoothed log returns into unsmoothed regular returns. We take this approach to assure returns are always above -100%, which is not the case during the financial crisis if we directly unsmooth regular returns. For robustness, Table IA.2 replicates Table 5 in the main text after relying only on regular returns. Results are similar, with the exposure to the public real estate market increasing from 0.04 to 0.67.

References for Internet Appendix

Fung, William, and David A. Hsieh. 2001. "The Risk in Hedge Fund Strategies: Theory and Evidence from Trend Followers". *Review of Financial Studies* 14 (2): 313–341.

Geltner, David. 1993. "Estimating Market Values from Appraised Values without Assuming an Efficient Market". *Journal of Real Estate Research* 8 (3): 325–345.

Getmansky, Mila, Andrew W. Lo, and Igor Makarov. 2004. "An econometric model of serial correlation and illiquidity in hedge fund returns". *Journal of Financial Economics* 74:529–609.

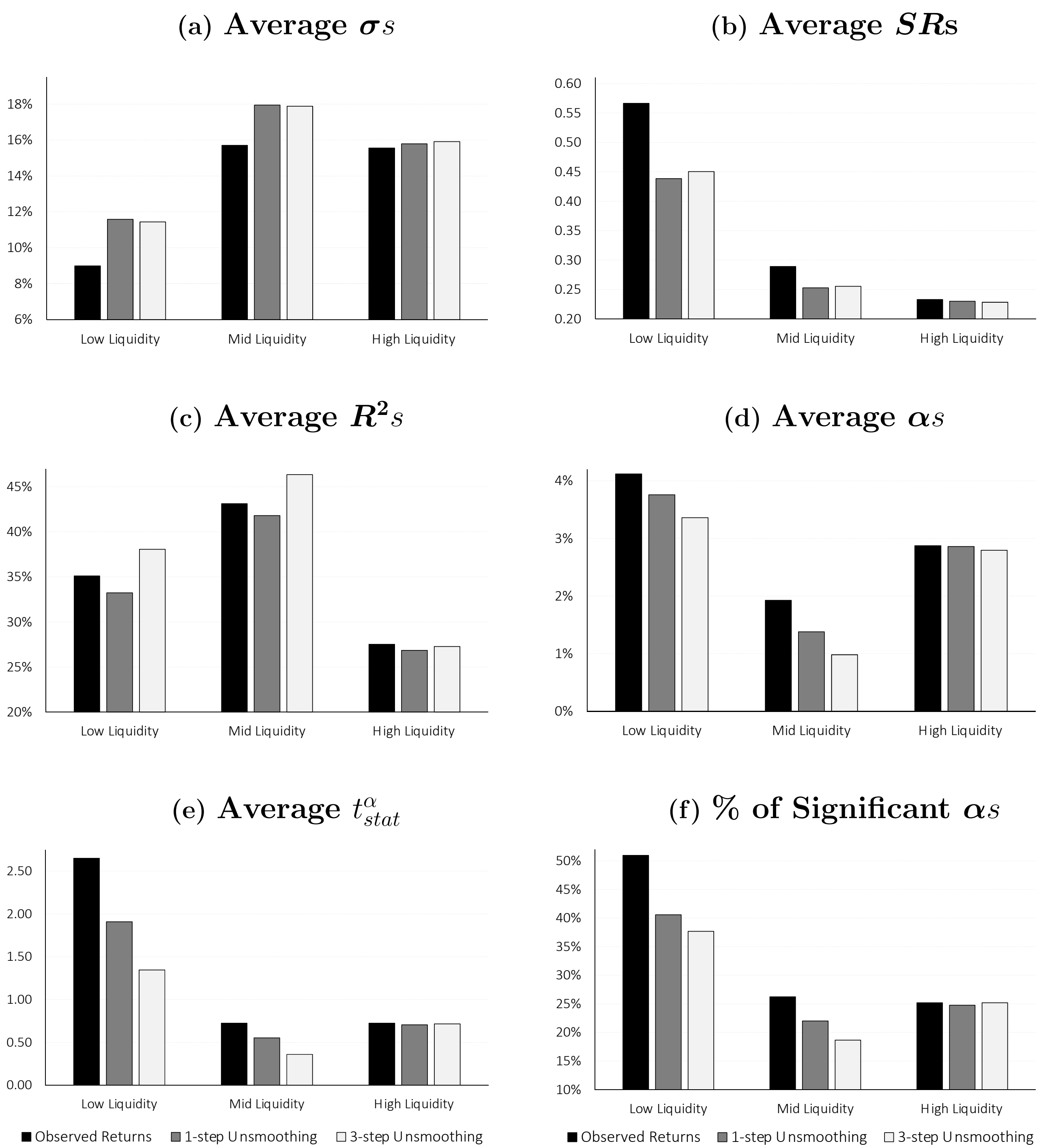


Figure IA.1
Hedge Fund Risk and Performance by Strategy Liquidity (Value-weights)

The figure plots average fund-level information for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2016. See Section 1 for the unsmoothing methods and Section 2.1 for further empirical details.

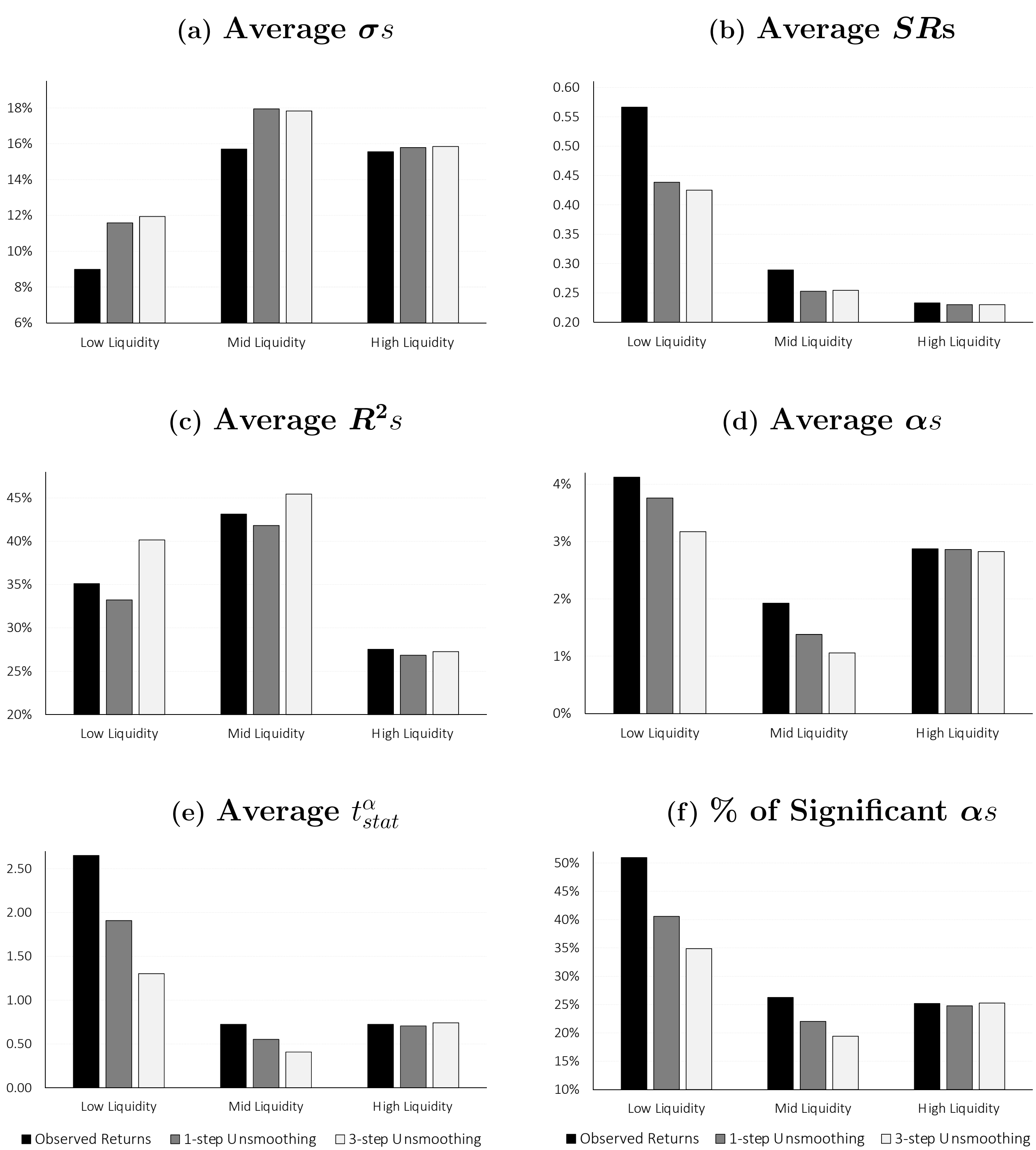


Figure IA.2

Hedge Fund Risk and Performance by Strategy Liquidity ($\pi_j^{(h)} = \overline{\pi}^{(h)}$)

The figure plots average fund-level information for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). $R^2 s$ and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2016. See Section 1 for the unsmoothing methods and Section 2.1 for further empirical details.

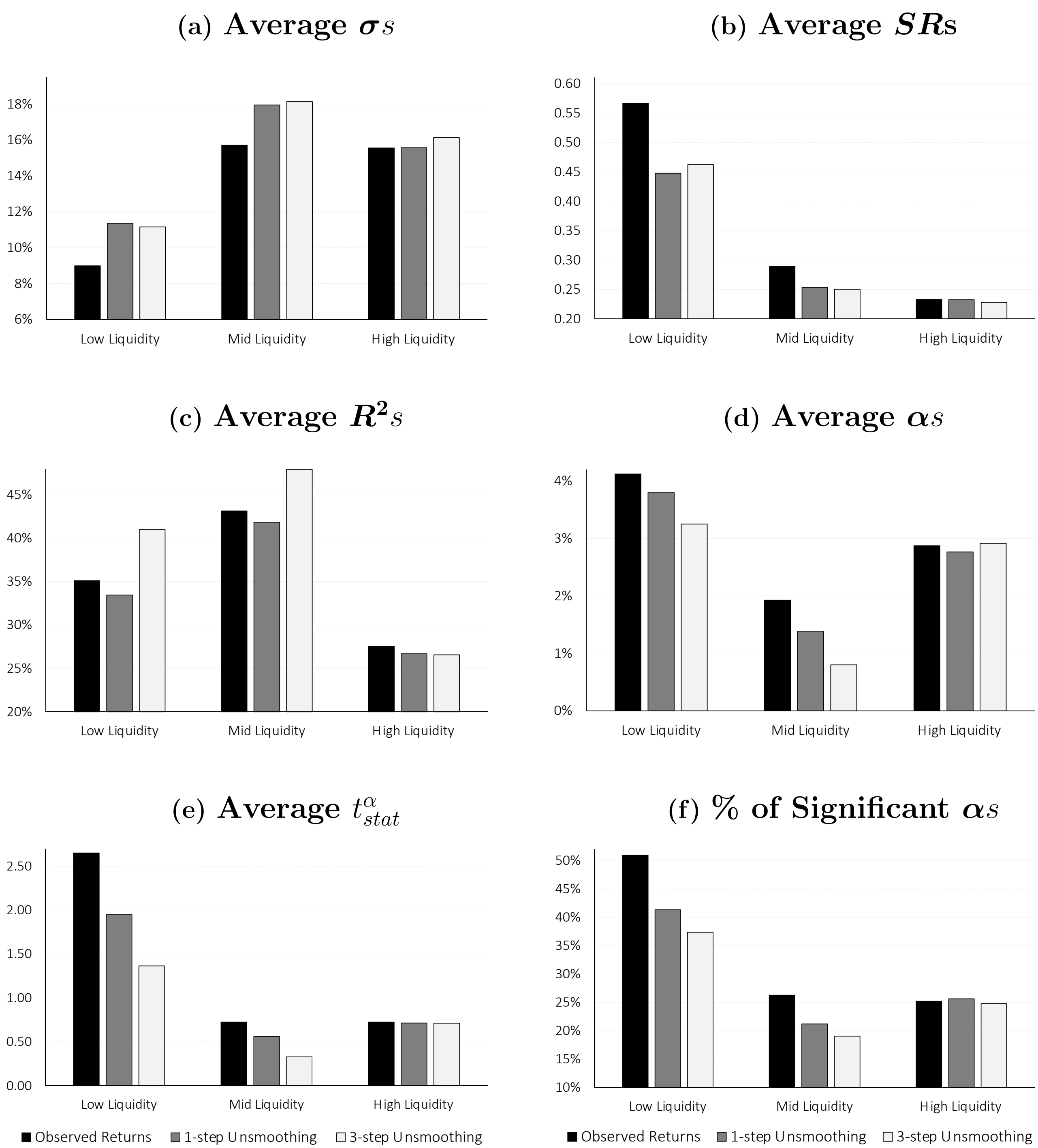


Figure IA.3
Hedge Fund Risk and Performance by Strategy Liquidity (H=2)

The figure plots average fund-level information for three groups based on hedge fund strategy liquidity using observed returns, 1-step unsmoothed returns (as in Getmansky, Lo, and Makarov (2004)), and 3-step unsmoothed returns. We sort strategies based on their first order autocorrelation coefficient to form three groups: low liquidity strategies (the three strategies with autocorrelation above 0.40), high liquidity strategies (the two strategies with autocorrelation below 0.10), and mid liquidity strategies (the other five strategies). R^2s and αs are based on the FH 8-Factor model that builds on Fung and Hsieh (2001) and statistical significance for fund-level αs is at 10%. The sample goes from January 1995 to December 2016. See Section 1 for the unsmoothing methods and Section 2.1 for further empirical details.

Table IA.1
Risk and Performance of OPRE Funds (Value-weights)

The table reports (average fund-level) statistics related to the risk and performance of US Open-ended Private Real Estate (OPRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from 1994 through 2017 and is restricted to OPRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Section 3.1 for the AR unsmoothing methods used and Section 3.2 for further empirical details.

	Returns	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	R^2	α	β_e	β_{re}
Statistic	Observed, R_o	5.5%	7.9%	0.70	4.0%	4.9%	0.03	0.04
						(82.8%)	(0.0%)	(17.2%)
	1-step, R_{1s}	5.5%	21.8%	0.25	14.8%	2.1%	0.05	0.30
						(10.3%)	(0.0%)	(65.5%)
	3-step, R_{3s}	5.5%	23.7%	0.23	25.9%	-0.6%	0.15	0.49
						(0.0%)	(0.0%)	(86.2%)
Change in Statistic	R_o to R_{1s}	0.0%	13.9%	-0.45	10.8%	-2.8%	0.02	0.26
						[-7.94]	[0.57]	[6.68]
	R_{1s} to R_{3s}	0.0%	1.9%	-0.02	11.2%	-2.7%	0.10	0.18
						[-6.29]	[2.58]	[4.90]
	R_o to R_{3s}	0.0%	15.8%	-0.47	21.9%	-5.5%	0.12	0.45
						[-13.08]	[7.57]	[15.42]

Table IA.2
Risk and Performance of OPRE Funds ($\pi_j = \overline{\pi}$)

The table reports (average fund-level) statistics related to the risk and performance of US Open-ended Private Real Estate (OPRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from 1994 through 2017 and is restricted to OPRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Section 3.1 for the AR unsmoothing methods used and Section 3.2 for further empirical details.

	Returns	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	R^2	α	β_e	β_{re}
Statistic	Observed, R_o	5.5%	7.9%	0.70	4.0%	4.9%	0.03	0.04
						(82.8%)	(0.0%)	(17.2%)
	1-step, R_{1s}	5.5%	21.8%	0.25	14.8%	2.1%	0.05	0.30
						(10.3%)	(0.0%)	(65.5%)
	3-step, R_{3s}	5.5%	25.8%	0.21	21.4%	-0.1%	0.01	0.55
						(0.0%)	(0.0%)	(89.7%)
Change in Statistic	R_o to R_{1s}	0.0%	13.9%	-0.45	10.8%	-2.8%	0.02	0.26
						[-7.94]	[0.57]	[6.68]
	R_{1s} to R_{3s}	0.0%	4.1%	-0.04	6.7%	-2.2%	-0.03	0.25
						[-7.91]	[-1.17]	[7.39]
	R_o to R_{3s}	0.0%	18.0%	-0.49	17.4%	-5.0%	-0.01	0.51
						[-20.81]	[-0.78]	[20.10]

Table IA.3
Risk and Performance of OPRE Funds (Regular Returns)

The table reports (average fund-level) statistics related to the risk and performance of US Open-ended Private Real Estate (OPRE) funds. All statistics are based on observed returns, 1-step unsmoothed returns (as in Geltner (1993)), and 3-step unsmoothed returns. The upper panel reports the values of the statistics (with the % of funds with significant values at 10% in parentheses) and the lower panel reports changes in these statistics (with the t_{stat} for a test of whether the mean change differs from zero in brackets). The sample goes from 1994 through 2017 and is restricted to OPRE funds that report return data to NCREIF and have at least 36 quarterly observations. See Section 3.1 for the AR unsmoothing methods used and Section 3.2 for further empirical details.

	Returns	$\mathbb{E}[r]$	σ	$\mathbb{E}[r]/\sigma$	R^2	α	β_e	β_{re}
Statistic	Observed, R_o	5.5%	7.9%	0.70	4.0%	4.9%	0.03	0.04
						(82.8%)	(0.0%)	(17.2%)
	1-step, R_{1s}	5.5%	21.8%	0.25	16.1%	1.6%	0.08	0.32
						(13.8%)	(0.0%)	(62.1%)
	3-step, R_{3s}	5.5%	30.8%	0.18	26.4%	-2.6%	0.16	0.67
						(0.0%)	(0.0%)	(93.1%)
Change in Statistic	R_o to R_{1s}	0.0%	13.9%	-0.45	12.0%	-3.3%	0.05	0.29
						[-8.11]	[1.71]	[7.42]
	R_{1s} to R_{3s}	0.0%	9.0%	-0.07	10.3%	-4.2%	0.08	0.34
						[-6.30]	[2.68]	[7.30]
	R_o to R_{3s}	0.0%	22.9%	-0.52	22.4%	-7.6%	0.14	0.63
						[-10.52]	[5.46]	[13.20]